

An Optimization Approach to Balancing Risk and Cost in Combatant Command Capability Advocacy

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Abstract: Unified Combatant Commands (UCCs) have broad continuing missions around the globe where they are tasked to provide functional expertise and defense of geographical areas. Accomplishing these missions requires a robust portfolio of military capabilities (e.g., aircraft, spacecraft, command and control systems, radar systems). UCCs routinely perform analyses to identify gaps between capabilities required to accomplish their mission and those currently at their disposal. Each year they submit a prioritized list of required capabilities, including new systems and greater capacity with existing systems, to the Joint Staff in the costly and time-consuming Integrated Priority List (IPL) process. This process relies on operational art and subject matter expertise, and sometimes fails to identify acquisition opportunities that achieve an optimal balance between risk and cost. Because this IPL process affects all of the DOD's personnel, material, systems and missions, it is arguably the most significant analytic challenge faced by the United States military. This article presents an integer linear programming model that computes an optimal balance between operational risk and the cost of acquiring new capabilities, and allows decision makers to identify the real-world impact of their budgetary decisions. We apply this model to the mission of providing aerospace defense of the United States and illustrate through sensitivity analysis the meaningful insights that can be gained by studying the relationship between the risk of not achieving 100 percent radar coverage and the opportunity cost of advocating for new capabilities.

Keywords: Advocacy Cost, Area Defense, Defensive Postures, Integer Linear Program

1. Introduction

1.1 Background

United States Department of Defense (DOD) Unified Combatant Commands (UCCs) exercise command authority of assigned forces in order to defend a specific area of interest and accomplish a broad continuing mission (Staff, 2013). Due to the breadth of their respective missions and the difficulty of being successful in a complex international landscape, UCCs must strike a balance between the risk of not being 100 percent successful against the high cost of acquiring the capabilities required to achieve 100 percent success. Fiscal realities dictate that commanders take on some amount of risk in this balance. UCCs advocate for greater capability via the Integrated Priority List (IPL), which is submitted annually to the United States military's Joint Staff and represents the prioritized issues that limit the command's ability to achieve its goals. Because this IPL process affects all of the DOD's personnel, material, systems and missions, it is arguably the most significant analytic challenge faced by the United States military.

United States Northern Command (USNORTHCOM) is one of the United States military's nine UCCs. USNORTHCOM is tasked to provide homeland defense efforts and to coordinate defense support of civil authorities (U.S.

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Northern Command, 2015). USNORTHCOM provides critical air, land, and sea defense to the United States, Canada, Mexico, and surrounding waters approximately out to 500 nautical miles off of the coast. In addition to providing international defense, USNORTHCOM's civil support mission includes domestic disaster relief operations. This support also includes counter-drug operations and pre-emptive terrorism management.

To accomplish its mission, USNORTHCOM, in conjunction with the North American Aerospace Defense Command (NORAD), requires a variety of weapons systems placed in strategic locations. However, each UCC experiences the need for weapons systems to accomplish its mission. Each time a commander requires a capability and wishes to move his/her desired capability to a higher priority on the IPL, they will incur an advocacy cost. For the purpose of this study, the term *advocacy cost* will represent the opportunity cost of placing a capability higher on the IPL than another.

1.2 Problem Statement

The decision maker (e.g., Commander of NORAD and USNORTHCOM) needs to know if the nation's current capabilities are adequate to provide aerospace defense of the United States. However, if these capabilities cannot protect the nation, the decision maker must be sure that any proposed capability will have a measurable, positive impact on the defense of the nation because he or she will be losing the opportunity to advocate for other, essential capabilities. Thus, the decision maker would like to balance an increase in the nation's survival, measured by both economic loss and citizen deaths, with an increase in advocacy cost, effectively minimizing some balance of risk and advocacy cost.

While this research focuses on the mission of aerospace defense, the broad goal of this project is to create a model that allows UCCs to quantify the risk associated with a given set of capabilities, determine the minimum cost solution that provides the capabilities needed to reduce or eliminate that risk, and strike a balance between risk and cost.

1.3 Related Work

The topic of resource allocation in defense acquisition has been studied in the recent literature. Griener et al. (2003) combines an analytic hierarchy process (AHP) analysis model with a binary integer portfolio optimization model into a decision support tool that aims to help the DOD in the planning, programming, and budgeting system. However, their proposed optimization model maximizes portfolio value in the presence of a constrained budget. Several other authors have studied portfolio optimization in the context of defense acquisition (see, e.g., Beaujon et al., 2001; Burk & Parnell, 2011; Dou et al., 2014; Salo et al., 2011). Though portfolio optimization has proven to be a valuable approach to allocating scarce resources in defense acquisition, our research aims to study the risk versus cost tradeoff to inform a decision for a particular capability.

Davis et al. (2008) takes a broad approach and presents a framework and methodology for DOD capability reviews. This approach uses decision analysis techniques to study a wide-ranging portfolio of defense options. While it is useful in determining capability gaps in a UCCs portfolio of capabilities, this approach does not address the aforementioned risk versus cost tradeoff specifically. Doing so requires optimization and sensitivity analysis, which the authors chose to omit.

Researchers have studied how to optimize placement of defensive systems for ballistic missile defense. Brown et al. (2005) presents a two-sided optimization model for prepositioning of defensive missile systems. An attacker-defender model is used to illuminate worst case scenarios. See Bertsekas et al. (2000), Diehl (2004), Liu et al. (2006), and Xing & Liu (2006) for other work focusing on optimizing missile defense system placement.

Lemay Center for Doctrine (2014) focuses on the Find Fix Track Target Engage Assess (F2T2EA) model, which is important for understanding how the aerospace defense process works. The find stage involves threat detection and requires commander guidance. The fix stage "identifies an emerging target as worthy of engagement and determines its position and other data with sufficient fidelity to permit engagement" (Lemay Center for Doctrine, 2014, p.2). Sensors focus on identifying the threat and its current location. The track stage "takes a confirmed target and its location, maintaining a continuous track" (Lemay Center for Doctrine, 2014, p.3). In the target stage, it "takes an identified, classified, located, and prioritized target; determines the desired effect and targeting solution against it; and obtains required approval to engage" (Lemay Center for Doctrine, 2014, p.3). During this phase, the following must take place: review of target restrictions, validation of effects and weaponeering capabilities, and analysis of collateral damage estimations. In the engage stage, "identification of the target as hostile is confirmed and engagement is ordered and transmitted to the pilot, aircrew, or operator of the selected weapon system" (Lemay Center for Doctrine, 2014, p.3). This stage requires all actions that will make engaging the target successful to occur. In the assess stage, "predetermined assessment requests are measured against actions and desired effects on the target" (Lemay Center for Doctrine, 2014, p.4). If the model was unsuccessful, a re-attack is required. The F2T2EA model will be utilized within this study by focusing on the Find and Engage pieces. The model will ensure that the defensive capabilities can detect the threat within a reasonable time to allow other defensive assets to intercept it.

Wilkening (1999) uses an analytic approach and treats defense as a Bernoulli trial problem. The equations used were the number of warheads, the probability warhead gets through defense, the conditional probability that the defense shoots down the warhead, and the probability that attacking warheads (or missiles) will penetrate the defense is given by the binomial distribution. Hit-to-Kill interceptors are a countermeasure to make sure they are a valid option. The different categories of countermeasures are circumvention, defense suppression, saturations and qualitative improvements in the offense. Although one could model air defense as a large system of Bernoulli trial problems, as we will see in Section 2, our study uses an integer linear program in order to focus on the tradeoff between risk and cost.

1.4 Contribution and Organization

The major contribution of this work, aside from developing a decision support tool for the specific mission of aerospace defense, is to demonstrate that a simple optimization model can be used to support high-level decision makers with one of the DOD's most significant challenges.

The remainder of this article is organized as follows. Section 2 describes our risk versus cost model and data collection. Section 3 presents numerical results of our model applied to the aerospace defense mission. Section 4 concludes with recommendations and comments about extending this work for other combatant commands.

2. Methodology

2.1 Risk versus Cost Model Formulation

We begin with a general model that could be used to inform the IPL process for any UCC. We suppose that each mission task t is valued by the UCC at some value v_t . Without loss of generality a mission task could refer to any particular UCC mission component that must be accomplished. We suppose that at least r_t units of capability are required to accomplish task t . The Generalized Risk Versus Cost Model (GRVCM) (1-4) is obtained by minimizing a linearly weighted balance of risk and cost, which is weighted by chosen parameter α . The left term in (1) calculates the risk of not fully accomplishing a task, while the right term in (1) calculates the cost of increased capability. In GRVCM, we seek the optimal values of binary task accomplishment variable X_t , which takes value 1 when task t is accomplished and 0 otherwise, and continuous capability variable C_t , which captures the amount of capability assigned to task t . We assume cost is a linear function f of capability and that capability is bounded. Lastly, we use a "Big M" term M in (2) to allow the task accomplishment variable X_t to take on value 1 only when capability C_t meets the required level r_t .

$$\min_{X_t, C_t} \sum_t [v_t(1 - X_t) + \alpha f_t(C_t)] \quad \forall t \quad (1)$$

$$s.t. \quad C_t - r_t \geq -M(1 - X_t) \quad \forall t \quad (2)$$

$$c_{min} \leq C_t \leq c_{max} \quad \forall t \quad (3)$$

$$X_t \in \{0,1\} \quad \forall t \quad (4)$$

While this is a rather simple integer linear program, it is useful to note that its value rests with performing sensitivity analysis on the weighting parameter α . While the units of value v_t and cost f are chosen at the discretion of the decision maker, it is useful to think of them in terms of value points and dollars respectively. Furthermore, the units of α are points per dollar in order to directly relate cost to risk. The objective function (1) is expressed in units of value, which could be, for example, commander's priority ranking, a risk measurement, or a UCC-specific capability measurement like we will see in Section 2.2. The range of α is $[0, A]$, where A is a value that allows cost to dominate value v_t . Large values of α weigh cost more heavily, while small values of α weigh risk more heavily. This allows the decision maker to consider this important tradeoff in an analytical manner.

We now consider the mission of aerospace defense of the United States where our mission tasks t correspond to covering cities with radar. In order to do this, we must consider the geometry of these operations (Lemay Center for Doctrine, 2014). In Figure 1 we zoom in on an arbitrary potential target location and consider worst case positioning of the radar and defensive assets with respect to the target and incoming threat. We see that a radar site (orange circle) must have capability C_t , which is at least r_t in order for the closest defender (black star), with response time d_t , to intercept a threat incoming with speed s_t . Using this geometry, we are able to reformulate (2) as (5).

$$\frac{C_t - r_t}{s_t} - d_t \geq -M(1 - X_t) \quad \forall t \quad (5)$$

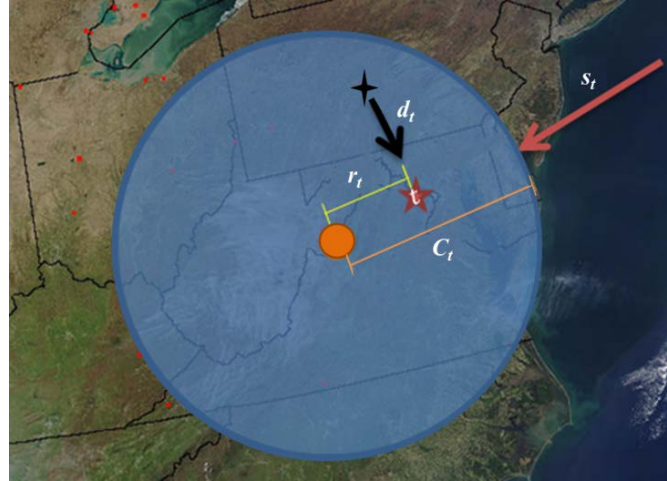


Figure 1. Depiction of Aerospace Defense Geometry

One could consider the more general case of (5) in order to account for arbitrary positioning of the radar and defensive asset with respect to the target and incoming threat, but doing so would result in a nonlinear and nonconvex constraint. Therefore, for model tractability we use the more conservative calculation.

In order to model the cost of new capabilities, we use advocacy cost. Advocacy cost is simply calculated as the capability gained over some initial capability i_t . This yields the cost function given in (6).

$$f_t(C_t) = C_t - i_t \quad \forall t \quad (6)$$

The final consideration that warrants discussion is handling capability that is common to a fleet of systems. New systems are procured as groups or fleets with like capability within the fleet. It may be unrealistic to assume that radars which monitor various targets can all possess the range tailored for each target. Rather, combatant commanders would likely upgrade systems in groups. This consideration can easily be modeled by a constraint (7) that requires the capability decision variable C_t for targets in the same group G to take on the same value.

$$C_t = C_{t'} \quad \forall t, t' \in G(t, t') \quad (7)$$

2.2 Aerospace Defense Data Collection

To best facilitate the analysis of various defensive postures to a variety of threats, the model requires data regarding offensive threats, defensive capabilities, and defense capability locations. Data for our model were collected from unclassified and open sources. They are intended to portray scenarios and capabilities representative of the aerospace defense mission. The model developed can be updated with new data without any difficulty.

Prior to collecting data, several simplifying assumptions were made. First, open source material concerning radar and defensive weapon locations were assumed to be accurate for use in the model. Second, the radar ranges given by the open source material were assumed to be the smallest accurate ranges, capable of detecting a missile head-on rather than when the missile is perpendicular to the radar. Third, only American-used equipment was considered. Fourth, different versions of various radars, aircrafts, and weapons were not considered (e.g., aircraft parameters were assumed to be the same across designations). Finally, each aircraft was assumed to be weaponized to its maximum capacity.

Military Periscope (2015) was utilized to collect data regarding defensive and offensive assets currently utilized in the United States. The necessary parameters were gathered based on the type of equipment; for example, the following data

were collected for aircraft: base location, combat radius, weapon and radar information (type, range and speed). In addition, open-source material concerning current radar and defensive weapon locations were collected. As an example of the type of data collected, Table 1 presents the data collected concerning F-15 defensive capabilities within the United States. These data would be used in the model to determine if a particular radar can locate a possible inbound threat and whether or not an F-15, at its current location and capability level, can intercept the attack on the United States before the attack reaches the target city. By completing this process for multiple defensive assets (aircraft, radar and missiles), and varying their locations and defensive capabilities within reason, the model will be able to produce a defensive posture that will decrease the risk of a catastrophic attack and decrease advocacy cost. For the sake of brevity and because further detail is unimportant to the exposition of our model, we omit data collected for other systems in this article. These data can easily be collected from Military Periscope (2015).

Table 1. F-15 Defensive Capabilities in the United States

Type	Location	Combat Radius (km)	Weapons Type	Weapons Name	Weapons Range	Radar	Radar Range (km)	Operational Limit Speed (km/h)
F-15	Nellis	1271	Cannon	1xM61A1	940 rds	1xAN/APG-63 (V)	370	2655
	Elmendorf		AAM	4xAIM-9 Sidewinder	8			
	Langley			4xAIM-7 Sparrow	45			
	Eglin			8xAIM-120 AMRAAM	50			

2.3 Determining Target Location

To determine where the United States’ enemies are likely to attack, a target list was created as a basis for attack and defense scenarios in the model. In order to create a target list that represents that of a notional enemy, the factors that may cause a certain location to be more favorable to attack than others were considered. By predicting what enemies might wish to gain from an attack on the United States, the following decision factors were used to objectively scale potential targets: Population, Gross Domestic Product (GDP), Strategic Importance, and Vulnerability. Population was included due to the assumption that enemies would attempt to strike a highly populated location in order to put fear and shock into a population. Each location’s population was included as a percentage of the maximum population in the model, New York City, and multiplied by a scaling weight of 10 to create 1-10 scale. The next characteristic, GDP, was similarly included due to the belief that potential enemies would target locations which have the ability to cripple the American economy. Again, this factor was included as a percentage of the maximum GDP and multiplied by a scaling weight of 6 to create a 1-6 scale. The final two characteristics, strategic importance and vulnerability, were given subjective values between 1 and 10 and multiplied by scaling weights of 8 and 1 respectively. Strategic importance was included because it is in the enemies’ best interest to focus their attacks on locations that give strong benefits to the power of the United States military forces. Finally, vulnerability was included with such a small weight because, given all other factors being equal, attacking the target with lesser perceived defensive capabilities will increase their probability of success. The goal of this work is to provide a decision support tool. Thus, since the representative data was used, the choice of weights here is not important. However, the model allows decision makers to alter these weights as appropriate. Summing the weighted decision variables yields (8).

$$TargetValue_i = 10*Population_i + 6*GDP_i + 8*StrategicImportance_i + 1*Vulnerability_i \tag{8}$$

Once (8) was determined, a short list of potential target cities in the United States was created based on predicted importance in each of the decision characteristics. For population and GDP, the largest cities in the United States were included, such as New York City, Los Angeles, and Chicago. For strategic importance, the cities that hold significant military importance were included, such as Washington D.C. Finally, for vulnerability, cities that appeared isolated, such as Honolulu, were included. These cities were combined into a potential target list and ranked in order of *TargetValue* using (8). This yielded an objective target list that shows each location’s overall significance in the eyes of the enemy. The resulting potential target locations are depicted in Figure 2. As the weighting system is linear, decision makers may use their

knowledge of current defenses to adjust the characteristic weights. We assume that distance from the center of the U. S. was appropriate to approximate vulnerability (Holmes et al., 2009). Based on subject matter expertise of defensive capabilities, users can alter our model to best represent the reality.

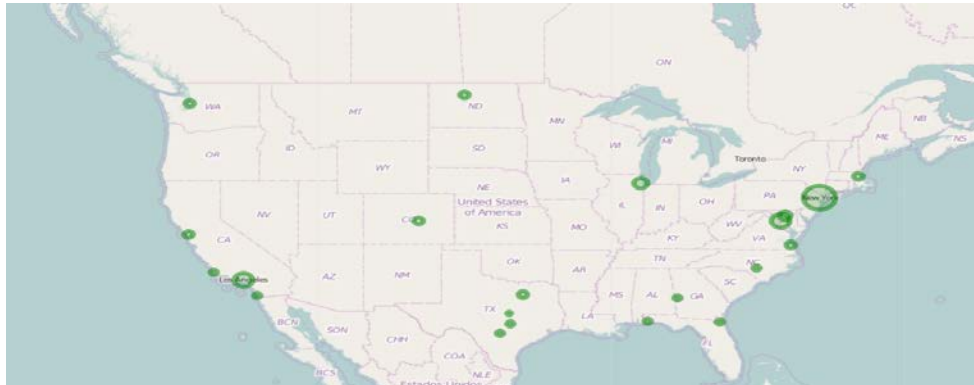


Figure 2. Depiction of Target List Locations. Larger Circle Radius Corresponds to Higher *TargetValue*.

3. Results and Analysis

3.1 Experiment Setup

Using the data collection process outlined in Section 2.2, we obtained a data set that consisted of 21 potential targets, 94 radar sites, each possessing one of seven radar types, and 15 defensive asset locations. Each potential target is covered by the closest radar site and the closest defensive asset. In order to model the worst case, incoming threats were assumed to travel the same speed as U.S. defensive assets. The data used in this experiment is given in Table 2, where all distances are in miles and times are in hours.

Table 2. Potential Target Data for Model Experiment

Potential Target	i [miles]	D [hours]	v [TargetValue]	r [miles]
1	229.91	0.67	208.63	673.44
2	242.34	0.17	387.31	171.15
3	229.91	0.51	261.42	511.77
4	198.84	0.65	428.93	651.33
5	198.84	0.1	326.9	103.63
6	198.84	0.23	197.46	230.52
7	198.84	0.58	302.03	586.14
8	198.84	0.14	198.48	143.35
9	229.91	0.24	211.17	241.45
10	229.91	0.35	236.04	355.09
11	198.84	0.25	577.16	251.76
12	229.91	0.12	256.35	122.07
13	198.84	0.28	1000	281.56
14	229.91	0.03	346.7	25.6
15	229.91	0.14	239.59	139.81
16	229.91	0.61	238.58	609.38
17	229.91	0.29	233	292.75
18	229.91	0.46	288.83	463.27
19	229.91	0.63	290.36	636.5
20	198.84	0.62	204.06	623.09
21	242.34	0.12	581.73	121.99

These data were input into Microsoft Excel, and modeled and solved using the OpenSolver add-in. Furthermore, we developed a decision support tool that allows the decision maker to balance risk and capability. The user has the ability to

input and alter radar data, defense data, vary α , and define radar groups G . After running the model, the user can investigate further to determine the specific radar range needed for each radar closest to each target and identify the targets the model determined will not be covered.

In order to demonstrate the utility of this model, we chose to vary α along its full range of meaningful values, which is 0 to 1 due to the relationship between *TargetValue* and advocacy cost. We consider two radar group scenarios. In the first scenario we restrict changes in radar capability to groups according to existing radar types. We refer to this as the Fleet Scenario. In the second scenario we allow all radars to take on the capability required to protect its assigned targets, even if that means that all radars would have different ranges. We refer to this as the Individual Scenario.

3.2 Numerical Results

We expect that when we set α to zero that our model, in either the Fleet Scenario or the Individual Scenario, will incur a high advocacy cost in order to gain the capabilities required to cover all potential targets with radar. Conversely, we expect that when we set α to its upper limit of one, in either scenario, our model will yield the zero cost status quo solution that does not cover all potential targets with radar. Interesting model behavior is observed when analyzing results in the middle of the α range.

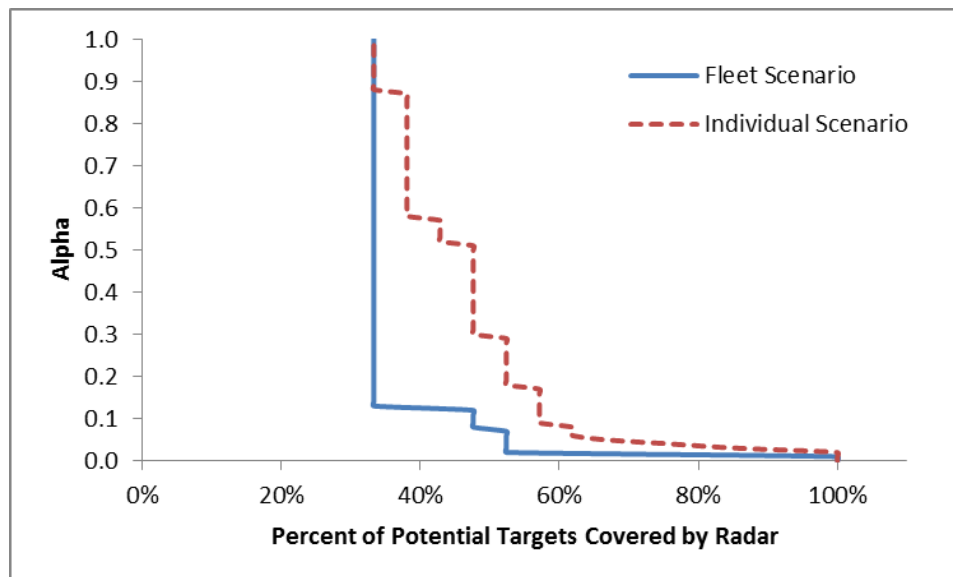


Figure 3. Coverage vs. α in the Fleet and Individual Scenarios

Figure 3 illustrates how the model behaves as α varies along its range, where the horizontal axis is a percentage of targets covered by radar out of the 21 total targets modeled. As expected, coverage is improved when the decision maker is risk averse. Interestingly, the Individual Scenario seems to outperform the Fleet Scenario. This can be explained by the fact that (7) restricts the model and yields solutions with higher objective function value. In operational terms, allowing the decision maker to increase capability only where needed, and not necessarily in groups of like systems, gives him/her the opportunity to cover more potential targets at the same cost. Unfortunately, the decision maker does not get to choose the fiscal scenario. However, understanding how this particular type fiscal constraint affects capability is important.

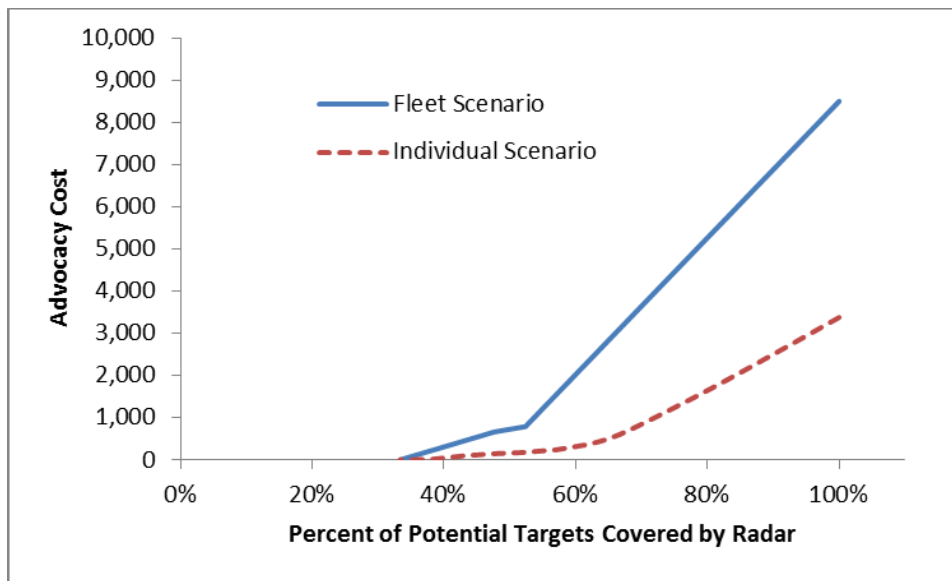


Figure 4. Coverage vs. Advocacy Cost in the Fleet and Individual Scenarios

Figure 4 illustrates how advocacy cost increases as an increasing percentage of targets are covered by radar. Recall that (6) dictates that advocacy cost is expressed in units of radar range. Given the specific form of the cost function f , one could easily convert advocacy cost into dollars. However, the behavior of these results would remain the same. Considering the cost difference in going from 48% coverage to 52% coverage in the Fleet Scenario, if, for example, radar capability costs \$50 thousand per mile (which is consistent with the cost of the ARSR-4; see Aftergood, 2000), then the marginal cost of ensuring radar coverage for potential target 9 (which corresponds to Jacksonville, Florida) is approximately \$6 million.

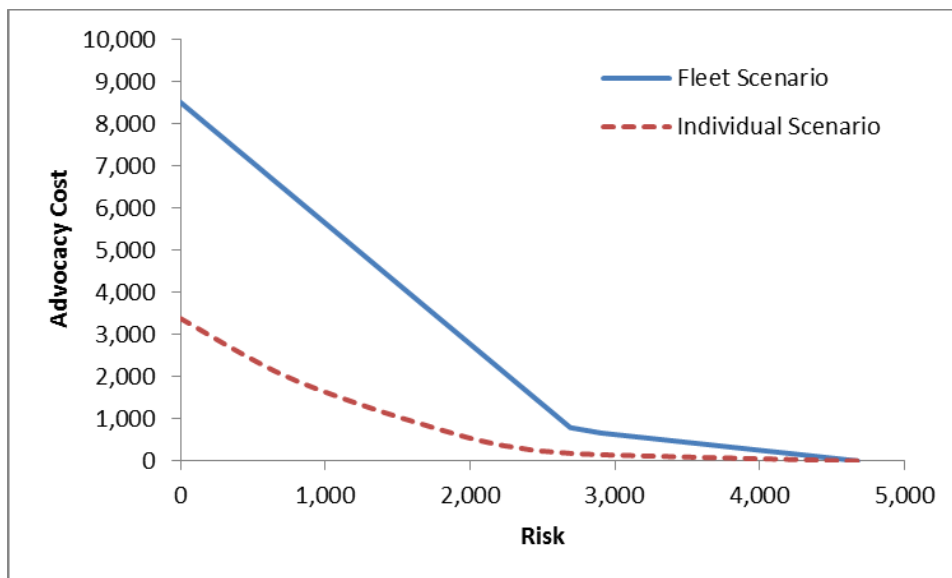


Figure 5. Risk vs. Advocacy Cost in the Fleet and Individual Scenarios

The information conveyed in Figure 4 is limited because it does not portray a complete picture of risk, which includes *TargetValue*. Figure 5 illustrates the relationship between risk and advocacy cost. Per (1) and (8), the units of risk

are the same as *TargetValue*. As the decision maker is able to pay more for increased capability, the risk of not covering valuable potential targets decreases. Observe that there appears to be a point of diminishing returns in each scenario near the risk threshold of 2,900. This is caused by high *TargetValues* of potential targets 11 and 13 (Los Angeles and New York respectively), combined with the distances to their respective defensive asset and radar locations. Insights like these could greatly assist UCCs as they weigh risk and cost in developing their respective IPLs.

3.3 Other Defense Applications

With appropriate data, this approach can be applied to a wide variety of defense procurement decision problems. For example, the United States Transportation Command (USTRANSCOM) is responsible for, among other things, providing global mobility to the DOD. USTRANSCOM could use this model to determine whether or not its airlift capability is sufficient to meet wartime requirements, and to evaluate which aircraft and basing strategy would best satisfy those requirements. In this context, the capability of interest would be expressed in terms of million ton-miles per day (MTM/D) and the parameter α would relate MTM/D to the costs of new aircraft and bases (or modernizing/maintaining existing aircraft and bases).

We could also consider UCCs that have a strong focus on Building Partnership Capacity (BPC), e.g., United States European Command and United States Southern Command. They could use the GRVCM to evaluate their ability to conduct joint exercises and training with partner nations in their respective regions. In this application, since BPC is a labor-intensive mission, the capability of interest could be expressed in terms of man-days in each of many functional areas (e.g., logistics, maintenance, medical, personnel) and the parameter α would relate man-days to the cost of increasing capability in each functional area.

4. Conclusions and Future Research

This article presented a novel approach to aid decision makers at UCCs to weigh risk and cost as they set priorities for the use of limited resources in acquiring new capabilities. This process often relies on operational art and subject matter expertise. Our proposed model is a useful alternative that facilitates data-driven decisions.

Numerical results focusing on the mission of aerospace defense illustrate how decision makers can balance risk and cost. Advocacy cost decreases as decision makers are willing to assume more risk. Our proposed model allows decision makers to see how these tradeoffs affect their mission in real terms, such as protecting cities in the United States.

The success of this work relies on UCCs to be compelled to make more data-driven decisions and to use a modeling approach like the one proposed as they advocate for the capabilities they need to accomplish their respective missions. An important extension of this work would be to further study DOD-level budget processes in an effort to find common measures for risk. Common risk measures would allow the DOD to develop an enterprise-level model that can balance risk and cost for all UCCs. This has the potential to altogether eliminate the costly and time-consuming IPL process.

With respect to the mission of aerospace defense, the decision space could be expanded to include the placement of defenses and radars as well as the utilization of probabilities of detection within the model, which may be accomplished by including new decision variables and including set covering constraints (Bates et al., 2015). Radars have varying probabilities of detection and it may be the case that several radars pick up a threat before the radar closest to each target. Therefore, future work may involve including these multiple-radar scenarios into the model while also incorporating probability of detection. Introducing these details into the analysis, however, would require a significant change to the GRVCM formulation.

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