Parameter Estimation of Type-I and Type-II Hybrid Censored Data from the Log-Logistic Distribution

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Abstract: In experiments on product lifetime and reliability testing, there are many practical situations in which researchers terminate the experiment and report the results before all items of the experiment fail because of time or cost consideration. The most common and popular censoring schemes are type-I and type-II censoring. In type-I censoring scheme, the termination time τ is pre-fixed, but the number of observed failures is a random variable. However, if the mean lifetime of experimental units is somewhat larger than the pre-fixed termination time, then far fewer failures would be observed and this is a significant disadvantage on the efficiency of inferential procedures. In type-II censoring scheme, however, the number of observed failures is pre-fixed, but the experiment time is a random variable. In this case, at least pre-specified R number of failure are obtained, but the termination time is clearly a disadvantage from the experimenter's point of view. To overcome some of the drawbacks in those schemes, the hybrid censoring scheme, which is a mixture of the conventional type-I and type-II censoring schemes, has received much attention in recent years. In this paper, we consider the analysis of type-I and type-II hybrid censored data where the lifetimes of items follow two-parameter log-logistic distribution. We present the maximum likelihood estimators of unknown parameters and asymptotic confidence intervals, and a simulation study is conducted to evaluate the proposed methods.

Keywords: Type-I Censoring, Type-II Censoring, Hybrid Censoring, Log-Logistic Distribution, Maximum Likelihood Estimation

1. Introduction

There has been a high demand in improving quality, productivity, and reliability of manufactured products over the last several decades. To meet this demand, manufacturers conduct appropriate designed experiments. In many cases when reliability or lifetime data are collected, all items in the experiment may not fail. Because of time or cost consideration, the practitioners terminate the experiment and report the results before all items realize their failures. The two most common censoring schemes are type-I and type-II censoring. In conventional type-I censoring, the failure is observed only if it occurs prior to some pre-specified time τ . In this instance, any failure that occurs after time τ is not observed. This type-I censored data are usually obtained when censoring time is fixed and the number of failures in that fixed time is a random variable. Such an experiment may save time and money because it could take a very long time for all items to fail. Another censoring often used in testing of equipment is type-II censoring where all items are put on test at the same time and the test is terminated when the pre-determined R number of the items have failed. In type-II censoring, the number of observed failures is fixed and the terminating time is random. It is true that the statistical treatment of type-II censored data is simpler because the data consists of the R smallest lifetimes in a random sample of lifetimes so that the theory of order statistics is directly applicable to determining the likelihood and any inferential technique employed.

Epstein (1954) first introduced the hybrid censoring scheme which is a mixture of type-I and type-II censoring schemes. Two types of hybrid censoring have been introduced. Let us consider n items and denote the ordered failure times of the items as $x_{(1)} < x_{(2)} < \cdots < x_{(n)}$. In type-I hybrid censoring, the experiment continues until a pre-determined time τ has been reached or a pre-specified number $R < n$, out of n items has failed. Thus the experiment is terminated at the random time $T = \min(x_{(R)}, \tau)$. The termination time here is at most T. Motivated by the work of Bartholomew (1963) and Barlow et al. (1968), Chen and Bhattacharya (1988) considered type-I hybrid censored data from an exponential distribution and provided an exact lower confidence band for a parameter. For more results on type-I hybrid censoring, one may refer to Ebrahimi (1992), Jeong et al. (1996), and Gupta and Kundu (1998) among others. Like conventional type-I censoring, the main disadvantage of the type-I hybrid censoring is that there may be few failures occurring up to the pre-determined time. In the type-II hybrid censoring, the termination time of experiment is $T = \max(x_{(R)}, \tau)$ which provides at least R failures observed up to the termination time. It may be noted that the complete, type-I and type-II censoring are special cases of hybrid censoring scheme by taking $R = n$ and $\tau \to \infty$. Hybrid censoring has become more popular and many authors have discussed statistical inference for various distributions under hybrid censoring scheme in the reliability literature; for example, Draper and Guttman (1987), Childs et al. (2003), Kundu (2007), Banerjee and Kundu (2008), Kundu and Pradhan (2009), Dube et al. (2010), and Balakrishnan and Kundu (2013), Asgharzadeh et al. (2013) among others.

Even though hybrid censored data have been discussed under some parametric lifetime distribution, two parameter hybrid censored log-logistic distribution has not been studied before. The log-logistic distribution is a good reliability model as it fits well in many practical situations of reliability data analyses. For example, Chiodo and Mazzanti (2004) discussed the log-logistic distribution for describing the degradation rate for highly reliable products, Kantam et al. (2006) used the log-logistic distribution for the basic probability model of the life of the product, and Akhtar and Khan (2014) utilized log-logistic distribution as a reliability model using a Bayesian method. Another important feature with the log-logistic distribution is that its reliability and hazard functions can be written in closed forms. Thus the log-logistic distribution is convenient in handling censored data. Hence, in this paper we consider the analysis of the hybrid censored data where the lifetime of items follows the log-logistic distribution.

The rest of this paper is organized as follows. Section 2 introduces the model assumptions and the maximum likelihood estimators of underlying parameters from hybrid censored log-logistic distribution. Section 3 contains simulation results to evaluate the performance of the estimators based on the proposed censoring schemes. Finally, we conclude the paper in Section 4.

2. Model and Maximum Likelihood Estimation

2.1 Model

Suppose that the lifetime X of a test item follows a log-logistic distribution of a shape parameter α and a scale parameter λ with the probability density function

$$f(x) = \frac{\alpha x^{\alpha-1}\lambda}{\left(1+\lambda x^{\alpha}\right)^2}, \quad x \ge 0, \alpha > 0, \lambda > 0 \tag{1}$$

Its corresponding cumulative density function, reliability function, and hazard function are given, respectively, by

$$F(x) = \Pr(X \le x) = \frac{\lambda x^{\alpha}}{1+\lambda x^{\alpha}} \tag{2}$$

$$S(x) = \Pr(X > x) = \frac{1}{1+\lambda x^{\alpha}} \quad (3)$$

$$h(x) = f(x)/S(x) = \frac{\alpha x^{\alpha-1}\lambda}{1+\lambda x^{\alpha}} \quad (4)$$

As described, the log-logistic distribution has closed forms of reliability and hazard functions. The following is an example of log-logistic distribution with $\lambda = 1$ and various α values.

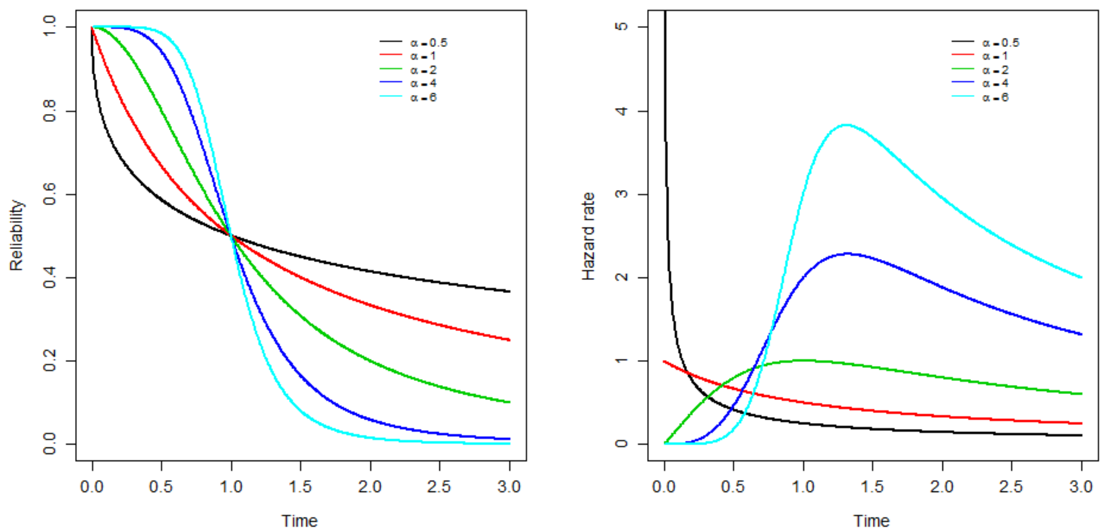

Figure 1. Reliability and Hazard Functions of Log-Logistic Distribution with $\lambda = 1$ and Various α Values

The log-logistic distribution is the probability distribution of a non-negative random variable whose logarithm has a logistic distribution. Its shape is similar to the log-normal distribution, but has heavier tails. The most commonly used distribution in the modelling of reliability and failure time data is the Weibull distribution. However, its use is limited by the fact that its hazard function must be monotonic while it may be increasing or decreasing. This may be inappropriate in a situation when failure reaches a peak after some finite period and then declines slowly. The log-logistic distribution can be used in such a case.

2.2 Maximum Likelihood Estimator

Let $x_1, x_2, \cdots x_n$ be n independent and identically distributed sample from two parameter log-logistic random variable X. Suppose $x_{(1)} < x_{(2)} < \cdots < x_{(n)}$ denote the ordered $x_1, x_2, \cdots x_n$. First, we will study type-II hybrid censoring scheme. Under the type-II hybrid censoring scheme, R and τ are known in advance and the termination time of experiment is $T = \max(x_{(R)}, \tau)$. As mentioned in Childs et al. (2003), type-II hybrid censoring may arise in a situation when R number of failure occur before the pre-specified time τ then the experiment can continue to make full use of the testing facility up to time τ. If the R failures do not occur before time τ, then the experiment continues until R failures. So we have one of the two following types of observations.

Case I: $\{x_{(1)} < x_{(2)} < \cdots < x_{(R)}\}$ when $x_{(R)} > \tau$ and pre-specified R number of failure occurred after the pre-specified censoring time τ.

Case II: $\{x_{(1)} < x_{(2)} < \cdots < x_{(k)}\}$ when $R \le k \le n, x_{(k)} < \tau < x_{(k+1)}$ and pre-specified R number of failure occurred and experiment continues until the censoring time τ. Here k is the number of failures observed before time τ.

To simplify the notations, we use x_i in place of $x_{(i)}$ hereafter. Then, the likelihood function for Case I is

$$L(\alpha,\lambda) \propto \prod_{i=1}^{R} \frac{\alpha x_i^{\alpha-1}\lambda}{(1+\lambda x_i^{\alpha})^2} \cdot \left(\frac{1}{1+\lambda x_R^{\alpha}}\right)^{n-R} \tag{5}$$

and the likelihood function for Case II is

$$L(\alpha,\lambda) \propto \prod_{i=1}^{k} \frac{\alpha x_i^{\alpha-1}\lambda}{(1+\lambda x_i^{\alpha})^2} \cdot \left(\frac{1}{1+\lambda \tau^{\alpha}}\right)^{n-k} \tag{6}$$

To obtain the maximum likelihood estimates (MLE), the natural logarithm of the likelihood function is usually considered. Combining the two cases, the log likelihood function can be written as

$$l = \sum_{i=1}^{k} \left\{\ln\alpha + (\alpha-1)\ln x_i + \ln\lambda - 2\cdot\ln\left(1+\lambda x_i^{\alpha}\right)\right\} - (n-k)\ln\left(1+\lambda c^{\alpha}\right) \tag{7}$$

where $c = x_R, k = R$ for Case I and $c = \tau, R \le k \le n$ for Case II. Then the maximum likelihood estimates of the parameters α and λ are solutions to the system of likelihood equations obtained by

$$\frac{\partial l}{\partial \alpha} = \frac{k}{\alpha} + \sum_{i=1}^{k} \ln x_i - 2\sum_{i=1}^{k} \frac{\lambda x_i^{\alpha}}{1+\lambda x_i^{\alpha}} \cdot \ln x_i - (n-k)\cdot \frac{\lambda c^{\alpha}}{1+\lambda c^{\alpha}} \cdot \ln c = 0 \tag{8}$$

$$\frac{\partial l}{\partial \lambda} = \frac{k}{\lambda} - 2\sum_{i=1}^{k} \frac{\lambda x_i^{\alpha}}{1+\lambda x_i^{\alpha}} - (n-k)\cdot \frac{c^{\alpha}}{1+\lambda c^{\alpha}} = 0 \tag{9}$$

Here, it is difficult to obtain a closed form solution to nonlinear score equations, so an iterative method such as the Newton-Raphson method is used to solve the equations to obtain MLEs. The MLE has asymptotic variance of the inverse of Fisher information matrix. Since the MLEs of parameters are not obtained in closed forms, it is not possible to obtain the Fisher information matrix and construct asymptotic confidence intervals. So the Fisher information will be approximated by the observed Fisher information evaluated at the MLE. The asymptotic confidence intervals based on the asymptotic normal distribution of MLEs are approximated by the inverse of observed Fisher information matrix evaluated at the MLE

$$\Sigma = \begin{bmatrix} \dfrac{\partial^2 l}{\partial \alpha^2} & \dfrac{\partial^2 l}{\partial \alpha \partial \lambda} \\ \dfrac{\partial^2 l}{\partial \lambda \partial \alpha} & \dfrac{\partial^2 l}{\partial \lambda^2} \end{bmatrix}^{-1} \tag{10}$$

where the elements of the observed Fisher information matrix are as follows:

$$\frac{\partial^2 l}{\partial \alpha^2} = -\frac{k}{\alpha^2} - 2\sum_{i=1}^{k} \frac{\lambda x_i^{\alpha}(\ln x_i)^2}{(1+\lambda x_i^{\alpha})^2} - (n-k)\cdot \frac{\lambda c^{\alpha}(\ln c)^2}{(1+\lambda c^{\alpha})^2} \tag{11}$$

$$\frac{\partial^2 l}{\partial\alpha\partial\lambda} = \frac{\partial^2 l}{\partial\lambda\partial\alpha} = -2\sum_{i=1}^{k}\frac{x_i^\alpha}{(1+\lambda x_i^\alpha)^2}\cdot\ln x_i - (n-k)\cdot\frac{c^\alpha}{(1+\lambda c^\alpha)^2}\cdot\ln c \quad (12)$$

$$\frac{\partial^2 l}{\partial\lambda^2} = -\frac{k}{\lambda^2} + 2\sum_{i=1}^{k}\left(\frac{x_i^\alpha}{1+\lambda x_i^\alpha}\right)^2 + (n-k)\cdot\left(\frac{c^\alpha}{1+\lambda c^\alpha}\right)^2 \quad (13)$$

Thus, an asymptotic confidence intervals for α and λ are obtained by

$$\hat{\alpha} \pm z_{c/2}\sqrt{\hat{\Sigma}_{11}} \text{ and } \hat{\lambda} \pm z_{c/2}\sqrt{\hat{\Sigma}_{22}} \quad (14)$$

where $z_{c/2}$ is the $100c$ upper percentage point of the standard normal distribution and $\hat{\Sigma}_{jj}$ is the (j, j) component of Σ evaluated with the MLE.

Now, let us consider type-I hybrid censored data, where the experiment is terminated at the random time $T = \min(x_{(R)}, \tau)$. Similarly, we have one of the two following types of observations:

Case I: $\{x_{(1)} < x_{(2)} < \cdots < x_{(R)}\}$ when $x_{(R)} < \tau$ and pre-specified R number of failure occurred before the pre-specified censoring time τ .

Case II: $\{x_{(1)} < x_{(2)} < \cdots < x_{(d)}\}$ when $x_{(d)} < \tau < x_{(d+1)}$ and only $d < R$ number of failure occurred before the pre-specified censoring time τ .

To simplify the notations, we use x_i in place of $x_{(i)}$ as before. Then, the likelihood function for Case I is

$$L(\alpha,\lambda) \propto \prod_{i=1}^{R}\frac{\alpha x_i^{\alpha-1}\lambda}{(1+\lambda x_i^\alpha)^2}\cdot\left(\frac{1}{1+\lambda x_R^\alpha}\right)^{n-R} \quad (15)$$

and the likelihood function for Case II is

$$L(\alpha,\lambda) \propto \prod_{i=1}^{d}\frac{\alpha x_i^{\alpha-1}\lambda}{(1+\lambda x_i^\alpha)^2}\cdot\left(\frac{1}{1+\lambda \tau^\alpha}\right)^{n-d} \quad (16)$$

Combining the two cases, thee log likelihood function can be written as

$$l = \sum_{i=1}^{r}\left\{\ln\alpha + (\alpha-1)\ln x_i + \ln\lambda - 2\cdot\ln\left(1+\lambda x_i^\alpha\right)\right\} - (n-r)\ln\left(1+\lambda c^\alpha\right) \quad (17)$$

where $c = x_R, r = R$ for Case I and $c = \tau, r = d$ for Case II. It must be noted that when $d = 0$ the maximum likelihood estimates do not exist. Because the log likelihood function is similar to in type-I hybrid censored data, the maximum likelihood estimates are obtained similarly by the method described in type-II hybrid censored data. Hyun et al. (2015) discussed this type-I hybrid censored data and the maximum likelihood estimators. For the work, one may refer to Hyun et al. (2015).

3. Simulation Study

The simulation study is performed for different choices of n and R . We take $\alpha = 2$ and $\lambda = 1$ in all cases. We set $\tau = 1$. For given parameters, we generate type-I and type-II hybrid censored samples from the log-logistic distribution. The results give the average failure percent, mean of the estimates (MLE), standard error of the estimates (SE), mean of the standard error estimates (SEE), and coverage probability (CP) of the proposed 95% confidence interval based on 1000 replications. For type-II hybrid censored data, the maximum follow-up time is provided in the parenthesis in Tables 3-6. We see that as the pre-specified number of failure increase, the follow-up increase to obtain the R failures. The MLEs of the parameters are obtained by solving the nonlinear equations through the Newton-Raphson iteration algorithm. Table 1 through Table 3 are summary statistics from type-I hybrid censored data, and Table 4 through Table 6 are from type-II censored data. From Tables 1-6, we observe that the biases and SEs decreases as the sample size increases and the failure rate increases. The standard error estimates provide a fairly accurate as SE and SSE are close. The standard error estimates provides a fairly accurate of true variance of the estimates, and the corresponding confidence intervals have reasonable coverage probabilities.

Table 1. Summary Statistics of Type-I Hybrid Censored Data with $n = 30$

	Failure %	Parameters	MLE	SE	SEE	CP
$R = 15$	46.5%	α	2.2207	0.5920	0.5262	94.8%
		λ	1.2007	0.6596	0.4845	94.1%
$R = 20$	50.1%	α	2.1325	0.5158	0.4884	95.1%
		λ	1.0978	0.4314	0.3998	94.1%
$R = 30$	50.2%	α	2.1288	0.5110	0.4872	95.1%
		λ	1.0917	0.4101	0.3962	94.1%

Table 2. Summary Statistics of Type-I Hybrid Censored Data with $n = 50$

	Failure %	Parameters	MLE	SE	SEE	CP
$R = 20$	39.8%	α	2.1792	0.4772	0.4420	95.4%
		λ	1.1862	0.5353	0.4146	95.3%
$R = 30$	49.9%	α	2.0771	0.3798	0.3683	95.0%
		λ	1.0607	0.3186	0.2978	95.0%
$R = 50$	50.2%	α	2.0724	0.3747	0.3667	95.2%
		λ	1.0546	0.3015	0.2945	94.7%

Table 3. Summary Statistics of Type-I Hybrid Censored Data with $n = 100$

	Failure %	Parameters	MLE	SE	SEE	CP
$R = 30$	30.0%	α	2.1238	0.3798	0.3595	95.4%
		λ	1.1495	0.4512	0.3516	96.1%
$R = 50$	48.0%	α	2.0499	0.2733	0.2627	94.9%
		λ	1.0394	0.2312	0.2107	94.8%
$R = 75$	50.0%	α	2.0320	0.2587	0.2544	95.0%
		λ	1.0228	0.2069	0.2011	94.5%
$R = 100$	50.0%	α	2.0320	0.2587	0.2544	95.0%
		λ	1.0228	0.2069	0.2011	94.5%

Table 4. Summary Statistics of Type-II Hybrid Censored Data with $n = 30$

	Failure %	Parameters	MLE	SE	SEE	CP
$R = 15$	53.7% (1.062)	α	2.1589	0.4931	0.4713	95.8%
		λ	1.1049	0.3986	0.3941	95.0%
$R = 20$	66.8% (1.386)	α	2.1492	0.4351	0.4093	95.4%
		λ	1.0988	0.3989	0.3696	94.3%
$R = 30$	100% (9.625)	α	2.0883	0.3350	0.3186	94.9%
		λ	1.0656	0.3628	0.3447	93.8%

Table 5. Summary Statistics of Type-II Hybrid Censored Data with $n = 50$

	Failure %	Parameters	MLE	SE	SEE	CP
$R = 20$	50.4% (1.004)	α	2.0761	0.3721	0.3660	95.4%
		λ	1.0560	0.2993	0.2945	95.2%
$R = 30$	50.3% (1.217)	α	2.0941	0.3399	0.3924	95.5%
		λ	1.0657	0.2943	0.2814	94.9%
$R = 50$	100% (12.384)	α	2.0521	0.2509	0.2426	94.9%
		λ	1.0411	0.2670	0.2575	94.7%

Table 6. Summary Statistics of Type-II Hybrid Censored Data with $n = 100$

	Failure %	Parameters	MLE	SE	SEE	CP
$R = 30$	50.0% (1.0)	α	2.0320	0.2587	0.2544	95.0%
		λ	1.0227	0.2069	0.2011	94.5%
$R = 50$	52.0% (1.038)	α	2.0440	0.2516	0.2493	95.1%
		λ	1.0289	0.2006	0.1995	95.0%
$R = 75$	75.0% (1.720)	α	2.0362	0.2040	0.1981	94.6%
		λ	1.0213	0.1863	0.1800	94.6%
$R = 100$	100% (18.214)	α	2.0257	0.1715	0.1694	95.0%
		λ	1.0171	0.1813	0.1770	94.5%

4. Conclusions

In this paper we considered the maximum likelihood estimators for the unknown parameters for type-I and type-II hybrid censored log-logistic distribution. From the simulation results it is found that the maximum likelihood estimates have good statistical properties. Although the lifetime distribution is assumed to follow log-logistic distribution, most of the methods in this paper can be applied to other lifetime distribution with hybrid censoring schemes. In type-I hybrid censoring model under which the experiment terminates as soon as either the first R failure or pre-specified censoring time τ occurs, the experiment can be terminated resulting in very few failures. For this reason, Childs et al. (2003), Chandrasekar et al. (2004), and Kundu and Joarder (2006) have focused on type-II hybrid censoring scheme under which the experiment terminate at the random time $T = \max(x_{(R)}, \tau)$. This type-II hybrid censoring scheme has the advantage of guaranteeing that at least R failures are observed. Extensive work has been performed on many variation of hybrid censoring as well. For some related work, one may refer to the work of Fairbanks et al. (1982), and progressive type-II censoring introduced by Cohen (1963).

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