Multiatributte Double Sampling Control Chart

Esmeralda Ramírez-Méndez and Mario Cantú-Sifuentes

Corporación Mexicana de Investigación en Materiales S.A. de C.V.
Calle Ciencia y Tecnología No. 790, Fracc. Saltillo 400,
C.P. 25290, Saltillo, Coahuila, México.

Corresponding author's Email: mcantu@comimsa.com

Abstract: In recent years, multiattribute control charts have received an increasing attention. These charts are able to monitor two or more attributes in the same chart. In addition, there are many applications of multiatributte control charts in a wide variety of manufacturing processes and services. In this article, a multiattribute double sampling (DS D2) control chart is proposed. Double sampling is a methodology used to improve the efficiency of a control chart to detect quality issues without increase the sampling. Results of comparative studies via simulation indicate that the proposed control chart significantly outperforms in most of the supposed sceneries, in terms of the Average Run Length.

Keywords: Average Run Length, Control chart, Multinomial.

1. Introduction

Nowadays, the statistical process control (SPC) is the most used tool to process control and to improve the product quality. The control chart is, perhaps, the main technique of the SPC, it is used to reduce the manufacturing processes variability. This reduction makes any company or industry more competitive.

A control chart is a graphical representation of a sequence of hypothesis tests. The null is formulated to establish that the process is *in control* (with respect to a reference) and the alternative hypothesis expresses that the process is *out of control*. To make the hypotheses test, in regular intervals of time, samples are taken from a production process and a test statistic is calculated. Each value is, sequentially, plotted and compared with two *control limits*; they define an arbitrary probability interval for the test statistic. Whereas, such a value falls within control limits, the process is declared in statistical control, otherwise, the process is declared *out of control*. This fact indicates that the process variability retains an amount of *variation* due to *especial* or *assignable causes*. On the other hand, there is a probability that out of control signal can be a false alarm; that is, even if the process is in control, signals out of control can be generated.

Control charts can be classified by type and number of quality characteristics to control in univariate and multivariate (continuous variables); uniattribute and multiattribute (discrete variables). Control charts for variables and attributes have been proposed in the literature. Recently, the last ones have received increasing attention; however, it is no comparable with the attention received for the variables control charts.

In 1924, Walter A. Shewhart introduced the first variable control chart. Since that time many approaches, generalizations and modifications have been proposed. For example, the adaptive control charts considers variable sample size or variable sample size frequency or both (De Magalhães et al., 2009; Mahadik and Shirke, 2011; Seif et al., 2011; Faraz et al., 2012); in the control charts with sequential sampling, the sampling is made in steps according to the location of the statistic values (Khoo et al., 2010; Irianto et al., 2010; Costa et al., 2011). There is another type of control chart named synthetic, which combines a classic control chart and the monitoring of a random variable, namely, the number of inspected samples among two out of control signals (Ghute and Shirke, 2008 and Khoo et al., 2013).

Generally, a production process is characterized for more than one quality attribute. So, for control purposes, it is necessary to monitor all attributes simultaneously. In literature, many strategies designed to monitor each attribute in separated control charts can be found. In this case, quality engineers must be able to monitor and interpret as many charts as attributes they consider. Moreover, there are approaches involving the simultaneous monitoring in just one graphic. Lu et al. (1998) developed a multiatributte control chart, named MNP, based on the binomial distribution. They defined the statistic X as the weighted sum of counts of nonconforming units of all quality characteristics in a sample. Jolayemi (1999) proposed a model to control multiatributte processes, which is an extension of univariate chart *np*. It is supposed that process attributes are independent binomial random variables.

Moreover, control charts that involve statistical and artificial intelligence techniques have been proposed by Taleb and Liman (2006) and Niaki and Nasaji (2011). A multiatributte synthetic control chart is proposed by Haridy et al., (2013).

There are processes where take large sample sizes is difficult, for example, the production standard is low, the inspection consist of destructive tests or, simply, to reduce the inspection cost. In particular, in a welding process, from automotive industry, a complete piece is classified in seven defects associated with welding, such a process has low volume of production daily. Consequently, this work addresses the problem to monitor multiatributte processes considering restrictions about sample size. In particular, a new control chart able to monitor simultaneously all of quality attributes in the same graphic is proposed. This chart is based on two methods: a multiatributte D^2 control chart, and on the other hand a double sampling method.

This paper is organized as follows: in the first section the introduction is presented, the methods description is given in the second section, which is divided in several subsections where, the base methods and proposed double sampling D^2 control chart are showed. In the third one, the results of a study of simulation are presented. Finally, the discussion and conclusions are given in the fourth section.

2. Method

2.1 D^2 Control Chart

The control chart proposed by Mukhopadhyay (2008) is able to monitor, simultaneously, more than one attribute. It is based on the generalized Mahalanobis distance and the multinomial distribution. This control chart is suitable to monitoring processes where a produced unit can be classified on several excluding nonconforming categories or defects.

Suppose that in a production process, a finished article can be classified in one and only one of k nonconforming categories including the conforming pieces. Let p_{ij} be the independent observed proportion of items with defect j, j=1, 2… k, in a sample of size N_i, so, $(p_{\text{i1}}, p_{\text{i2}}, p_{\text{i3}}, \dots, p_{\text{ik}}) = p_{\text{i}}^{\text{T}}$ is the vector of the proportions associated with each nonconforming category. Suppose $\bar{p}^T = [\bar{p}_1, \bar{p}_2, \dots, \bar{p}_k]$ denotes the vector of nonconforming categories of an in-control process. $\bar{p}^T$ can be estimated either by means of a historical data base, or arbitrarily specified. Then p_i^T has a multinomial distribution with parameters $\bar{p}^T$. In the rest of the paper $\bar{p}^T$ will be called the target proportions vector. It is clear that the target vector will be fixed to represent a production process having high quality: low proportions in nonconforming categories and high proportion in the conforming one.

For multinomial data, the generalized Mahalanobis distance is defined as:

$$D_i^2 = (p_i - \bar{p})^T \Sigma_i^{-1} (p_i - \bar{p}) = \sum_{j=1}^{k} \frac{N_i (p_{ij} - \bar{p}_j)^2}{\bar{p}_j} \quad (1)$$

Then, D_i^2 measures, in a particular point in time, the distance between the observed and the target proportions vectors. The arbitrary upper control limit (UCL) is

$$\text{UCL} = \left[\frac{(N_i(K-1))}{(N_i - K + 2)}\right] F_{K-1, N_i - K + 2, \alpha} \quad (2)$$

The quantity, $F_{K-1, N_i - K + 2, \alpha}$ are the quantile α of the F distribution with its respective $(K-1)$ and $N_i - K + 2$ degrees of freedom. The lower control limit is zero. See Mukhopadhyay (2008) for details.

2.2 Double Sampling

The double sampling (DS) is a particular case of sequential sampling, which consist in two sampling stages to decide whether a batch is accepted or rejected. Daudin (1992) was one of the first in study the double sampling as improvement alternative for variables control charts, he proposed the double sampling control chart for the mean (DS $- \bar{x}$).

The method proposed by Daudin presents more statistical efficiency in terms of Average Run Length (ARL) than the corresponding classical control chart, without increasing the expected overall sample size. Then, one can hypothesize that such a sampling scheme can be implemented in each control chart, in order to reduce the sampling size without reducing the chart statistical efficiency (He et al., 2002).

The double sampling method assumes that a sample of size $n_1 + n_2$ can be taken without difference of time, hence, all measurements comes from the same distribution. Observe the first n_1, in the first stage, and decide if the remaining n_2

must be observed. Thus, although the required time to inspection or measurement be long; the use of the double sampling is possible. Furthermore, is assumed that the samples are independent.

2.3 Description of the proposed Double Sampling D^2 Control Chart (DS D^2)

Suppose that a DS D^2 control chart is used to monitor a process where produced units can be classified in $K-1$ categories of defects and one conforming (or not defect) category, all of them mutually exclusive and exhaustive. As established, and in sense of these categories, the process quality can be specified in terms of the multinomial distribution with parameters n and $\bar{p}$. More items in nonconforming categories indicate worst process quality, and vice versa, more items in conforming category indicate best process quality. In terms of the Mahalanobis distance, whereas the $i-$th observed vector looks like the target vector the quantity D_i^2 must be near to cero, otherwise D_i^2 must be near to the upper control line.

The random variable to monitor is the number of units that are classified into K categories in a sample of size n and $\bar{p}$ represents the vector of proportions. The target value of $\bar{p}$ is represent by $\bar{p}_0$; this is, when the process is in control $\bar{p} = \bar{p}_0$. $\bar{p}_0$ can be estimated from historical data of the process or can be arbitrary specified.

As all of control charts, the purpose of the proposed chart is the efficiently detection of assignable causes of variation that result in a shift in $\bar{p}$, that is $\bar{p}$ changes to some $p_1 \neq p_0$. This shift is measured by means of the D^2 statistic defined in (1). D^2 is a generalized Mahalanobis distance that measures the difference between the vector target $\bar{p}_0$ and the vector of proportion observed from the process $\bar{p}$.

The DS D^2 control chart has five design parameters to set, the size of the first sample (n_1), the warning limit for the first sample (WL), the upper control limit for the first sample (UCL$_1$), the size of the second sample (n_2^*) and the upper control limit for the second stage (UCL$_2$). The lower limit is equal to zero.

Let d_1 and d_2^* the vectors containing the counts in the K categories of interest for first and second sample, respectively. Periodically, at fixed sampling intervals, a sample of size n_1 is drawn from the process. Then, by means inspection, d_1 is obtained to compute $\bar{p}$ (the vector of observed proportions) and the D_1^2 value. If D_1^2 < WL, the process is considered in control and the control scheme continues operating with sample size n_1. Otherwise, if D_1^2 > UCL$_1$, the process is supposed to be out of control and an investigation should be initiated. However, if WL < D_1^2 < UCL$_1$, an additional sample of size n_2 items is immediately taken. Now, the second sample of size n_2^* is inspected to obtain d_2* and D_2^2 is compute with $d_2 = (d_1 + d_2^*)$ and $n_2 = (n_1 + n_2^*)$. In this case, the decision depends on D_2^2 taken into account the information from the two samples. If D_2^2< UCL$_2$, the process is considered in control. But if D_2^2>UCL$_2$, the process is considered out of control and corrective actions should be taken. Whether the process is considered in control or is put back into control as a result of corrective actions, in the next sampling time scheduled it returns to the first phase of the DS scheme, taking a sample of size n_1.

Unlike D^2 control chart, for the DS D^2 the sample sizes n_1 and n_2 (Equal to$n_1 + n_2^*$) are supposed fixed. The statistics is as follows:

$$D_i^2 = n_i \sum_{j=1}^{k} \frac{(p_{ij} - p_{0j})^2}{\bar{p}_j} \quad (3)$$

Where p_{ij} is the observed proportion in the *i*-th sampling stage of the *j* category and $\bar{p}_0$ denoted the target vector. i = 1, 2 , j =1, 2,…, k and $\sum_{j=1}^{k} p_{ij} = 1$.

The warning limit and the upper control limit for the first sample and he upper control limit for the second stage is:

$$WA = \left[\frac{(n_1(K-1))}{(n_1 - K + 2)}\right] F_{K-1, n_1-K+2, \alpha_1} \quad (4)$$

$$UCL_1 = \left[\frac{(n_1(K-1))}{(n_1 - K + 2)}\right] F_{K-1, n_1-K+2, \alpha_2} \quad (5)$$

$$UCL_2 = \left[\frac{(n_2(K-1))}{(n_2 - K + 2)}\right] F_{K-1, n_2-K+2, \alpha_2} \quad (6)$$

Where $\alpha_1 > \alpha_2$ (one possible choice is to set $\alpha_1 = 0.05$ and $\alpha_2 = 0.01$). The quantities $F_{K-1, n_i-K+2, \alpha_i}$, are the quantile α_i of the F distribution with its respective $(K-1)$ and $(n_i - K + 2)$ degrees of freedom, (i = 1, 2).

3. Results

The performance of a control chart can be determined by the speed of detection of a shift or process disturbance that results in a quality loss. This speed can be measured in terms of the ARL.
The ARL is defined as the average number of samples taken until an out of control signal occurs. When the process is in control (i.e. $\bar{p} = \bar{p}_0$) is desirable that the ARL is large, this implies low false alarms rate. Moreover, when the process is out of control the ARL should be small to provide a fast detection of the process disturbance. It is supposed that process starts in control and that the shift does not occur while the sampling is doing, but between sampling intervals. Also is assumed that produced items are independent.

The objectives of this study are to know the performance of the control chart proposed in several sceneries and to determinate if the using of double sampling method improves its efficiency. For this purpose, a comparison of both control charts (DS D^2 and D^2) in terms of the ARL curves is done. The ARL curves were obtained by means of simulation.

In the simulation, the chart parameters were set as follows: $n_1 = 10, 30, 50, 70, 90$ and $n_2 = 20, 40, 60, 80, 100$. The number of categories to monitor was taken as k = 3, 6 and the desirable proportion of conforming units was assumed as $p_0 = 0.90, 0.95$ as two different sceneries and several cases were randomly generated from the multinomial distribution with parameter:

$$\hat{p}^T = \left[(p_0 - \rho), \frac{1 - p_0 + \rho}{k - 1}, \frac{1 - p_0 + \rho}{k - 1}, \frac{1 - p_0 + \rho}{k - 1}, \frac{1 - p_0 + \rho}{k - 1}, \frac{1 - p_0 + \rho}{k - 1}\right] \qquad (7)$$

Where ρ = 0.00, 0.01, ..., 0.4 are shift quantities in the proportions vector $\hat{p}^T$.

This study, basically investigates the efficiency, in likely monitoring sceneries, of the control chart to detect quality deterioration of the process, in ARL terms. The ARL curves for both charts, for the case k=3 and k=6, are showing in the Figure 1 and Figure 2, respectively. The y axis corresponds to log(ARL) instead ARL for presentation reasons and the x axis shows the ρ values. The Table 1 and Table shows ARL values only for the cases k=3 and k=6.

As can be observed from Figure 1 and Figure 2, the performance of both control charts are strongly affected for the sample size. That is, if the sample size decreases (in particular, n=10, red line) both control charts present the worst performance for both supposed sceneries. Despite, the efficiency of the DS D^2 for smaller samples size (n=10, 20) are significantly better and is slightly better than the D^2 chart where the sample size is bigger.

From Figure 1 (k=3) both control charts have an expected behavior. Nevertheless, from Figure 2 (k=6), we can observe that ARL curve of n=10 for (a) have erratic behavior, this can be interpreted as: D^2 control chart is unreliable to monitor such cases. Results indicate that both control charts have better performance when the difference between k and n is increase.

From simulation results in the Table 1 and Table 2, can be observed that in n=10 the ARL values of the proposed control chart are smaller than ARL values of D^2 control chart for $\rho \neq 0$, this means the DS D^2 control chart is more efficient detecting small to moderate shifts than the D^2 control chart when the sample size is 10, in ARL terms.

From comparing the columns $p_0 = 0.90$ and $p_0 = 0.95$ of Table 2, we can observed that both control charts have high ARL value for n=10 and $\rho = 0$, this means that rate of false alarms is low. Though, this value remains high for small values of ρ, this is no suitable for detection purposes. The results suggest that such situation happens for small sample size and small values of ρ (less than 0.03), and is worse for sceneries with high desirable proportion.

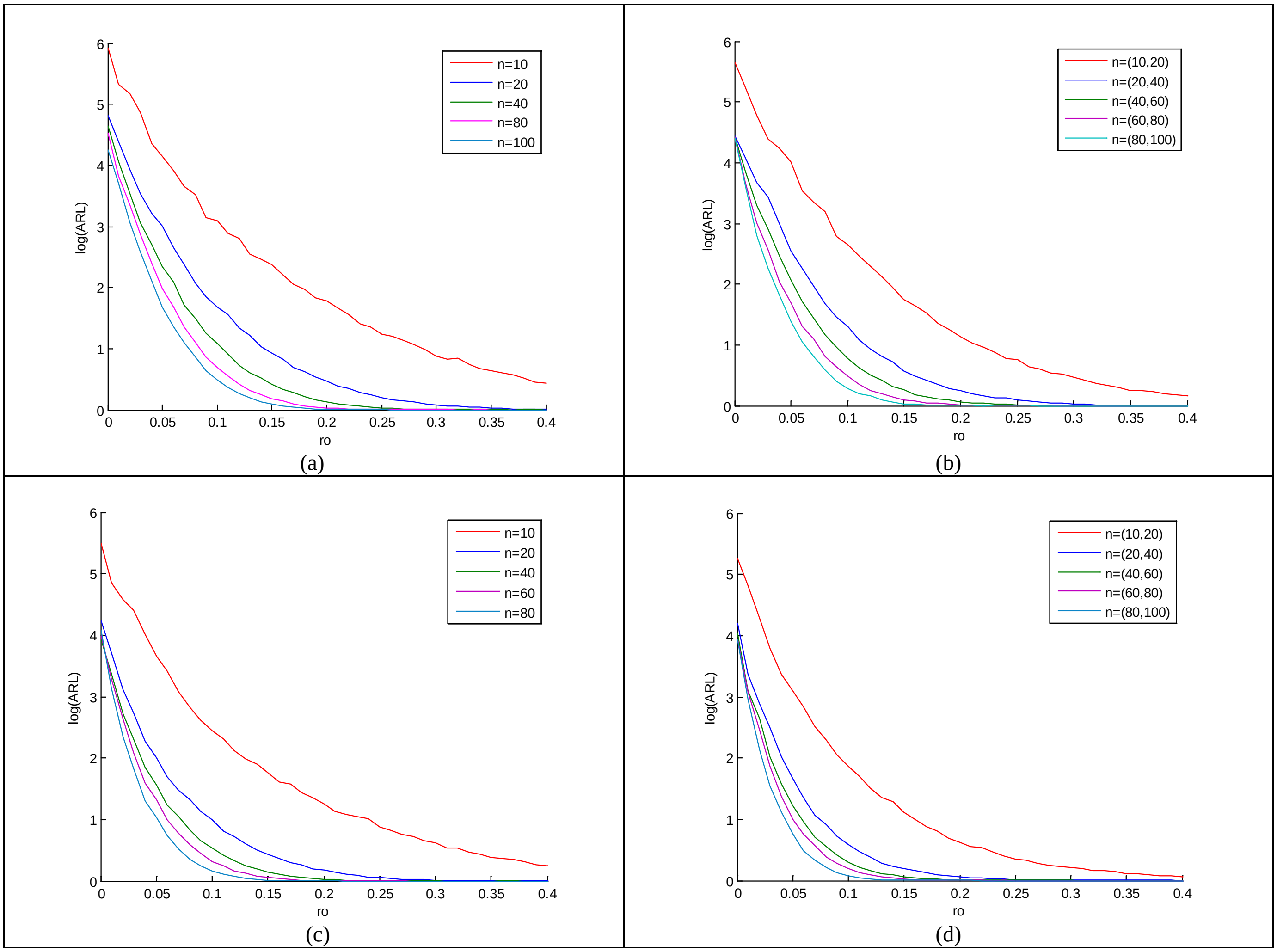

Figure 1. For k = 3, (a) D^2 control charts ARL curves and (b) DS D^2 control charts ARL curves, both with the desirable proportion of conforming units as 0.90; (c) D^2 control charts ARL curves and (d) DS D^2 control charts ARL curves, both with the desirable proportion of conforming units as 0.95.

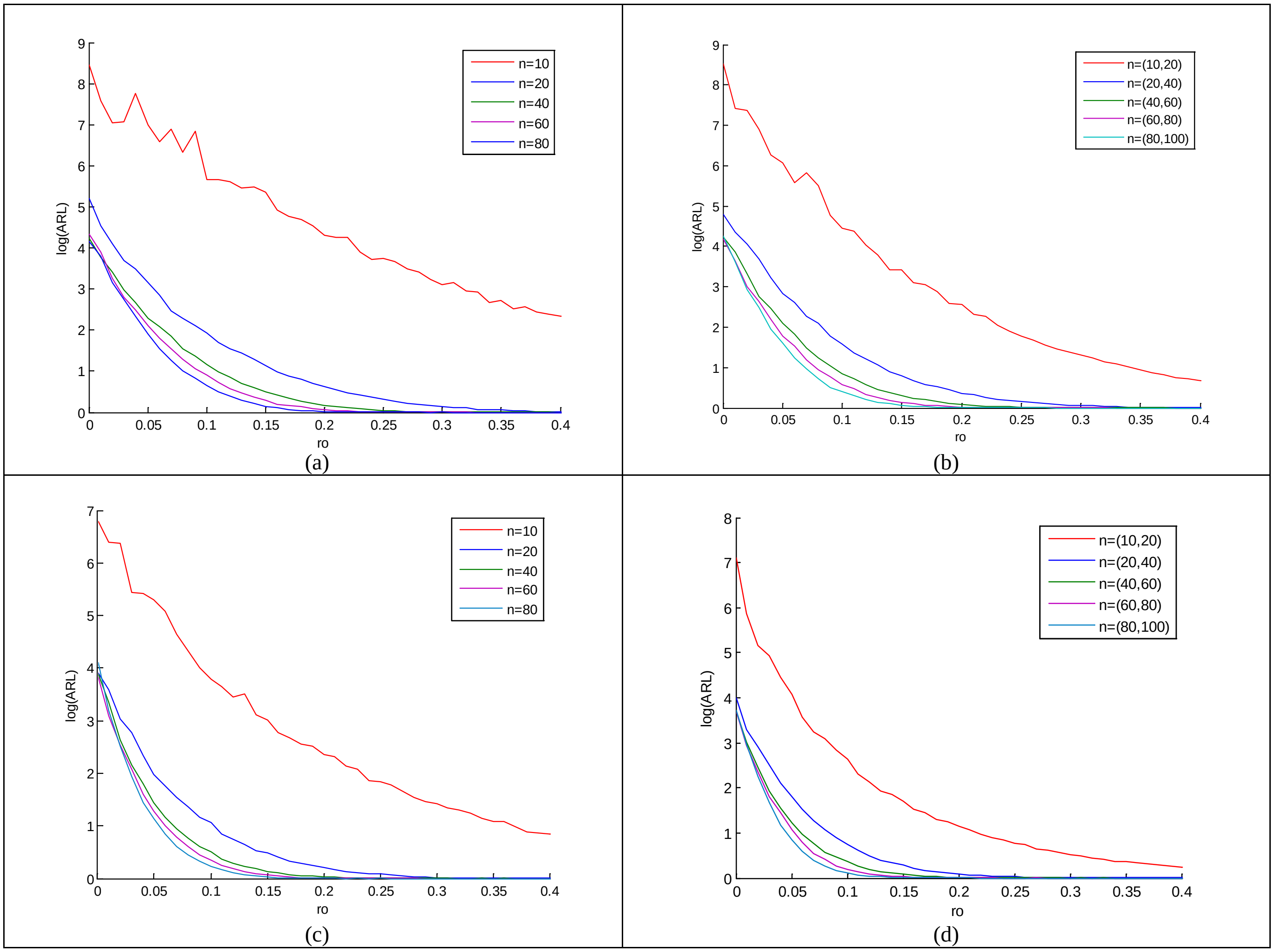

Figure 2. For k = 6, (a) D^2 control charts ARL curves and (b) DS D^2 control charts ARL curves, both with the desirable proportion of conforming units as 0.90; (c) D^2 control charts ARL curves and (d) DS D^2 control charts ARL curves, both with the desirable proportion of conforming units as 0.95.

From Table 1 and Table 2, both control charts have better performance for k=*3* than for k=6, in almost all simulated cases. Nonetheless, the proposed control chart is superior.

The proposed control chart has an evident advantage over D^2 control chart; this happen because of increasing of the sample size, raises the statistic accuracy, when the information in the first sample is insufficient to make a decision about of the process state. Although, the DS D^2 requires a second sampling, the cost of using the single sampling D^2 control chart is greater. This can be seen by the some simulated cases analysis. For example, in the case k=6, n=10, ρ=0.24, p_0 = 0.90 the D^2 control chart required, on average, 41.14 samples (410 items in all); while the DS D^2 control chart required 6.65 (at most 180 items, on average, assuming that carried out the two sampling stages). Then, it can be anticipated that, in practice, cost of using DS D^2 control chart will be lower.

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Table 1. ARL values for *k*=3.

		$p_0 = 0.90$		$p_0 = 0.95$	
n	ρ	ARL D^2	ARL DS D^2	ARL D^2	ARL DS D^2
10	0	372.92	283.47	243.85	192.04
	0.03	130.58	80.98	81.88	44.51
	0.06	50.37	34.26	30.72	17.19
	0.09	23.40	16.13	13.66	7.79
	0.12	16.53	9.96	8.33	4.53
	0.15	10.73	5.73	5.87	3.03
	0.18	7.19	3.87	4.23	2.24
	0.21	5.24	2.80	3.13	1.74
	0.24	3.87	2.18	2.76	1.49
	0.27	3.10	1.84	2.13	1.33
	0.3	2.41	1.59	1.87	1.25
20	0	123.21	85.40	69.39	67.28
	0.03	34.37	31.02	15.43	12.05
	0.06	14.16	9.58	5.46	3.90
	0.09	6.37	4.26	3.13	2.08
	0.12	3.82	2.53	2.06	1.47
	0.15	2.51	1.78	1.54	1.23
	0.18	1.85	1.41	1.29	1.09
	0.21	1.46	1.22	1.15	1.04
	0.24	1.28	1.13	1.07	1.02
	0.27	1.16	1.07	1.03	1.01
	0.3	1.08	1.03	1.01	1.00
50	0	104.93	83.37	50.57	56.51
	0.03	21.43	18.14	10.10	7.57
	0.06	8.07	5.54	3.43	2.61
	0.09	3.47	2.64	1.94	1.52
	0.12	2.08	1.65	1.39	1.18
	0.15	1.52	1.30	1.15	1.07
	0.18	1.24	1.12	1.06	1.02
	0.21	1.10	1.04	1.02	1.01
	0.24	1.04	1.02	1.01	1.00
	0.27	1.01	1.00	1.00	1.00
	0.3	1.01	1.00	1.00	1.00
70	0	91.58	82.88	54.29	51.41
	0.03	17.77	12.91	8.10	6.48
	0.06	5.34	3.67	2.73	2.13
	0.09	2.38	1.88	1.57	1.32
	0.12	1.53	1.29	1.18	1.10
	0.15	1.19	1.10	1.06	1.02
	0.18	1.07	1.04	1.02	1.01
	0.21	1.02	1.01	1.00	1.00
	0.24	1.00	1.00	1.00	1.00
	0.27	1.00	1.00	1.00	1.00
	0.3	1.00	1.00	1.00	1.00

Table 2: ARL values for *k*=6.

		$p_0 = 0.90$		$p_0 = 0.95$	
n	*ρ*	ARL D^2	ARL DS D^2	ARL D^2	ARL DS D^2
10	0	4707.00	>5000.00	881.50	1216.50
	0.03	1178.75	996.40	228.05	138.66
	0.06	724.00	266.50	159.87	35.96
	0.09	939.40	118.38	54.49	17.06
	0.12	275.50	56.01	31.69	8.51
	0.15	213.91	30.82	20.29	5.49
	0.18	108.59	17.68	12.98	3.71
	0.21	71.16	10.22	10.11	2.92
	0.24	41.14	6.65	6.38	2.32
	0.27	32.72	4.76	5.26	1.92
	0.3	22.61	3.67	4.17	1.69
20	0	181.12	120.28	49.44	54.09
	0.03	39.83	39.67	16.05	12.35
	0.06	17.43	13.75	5.81	4.66
	0.09	8.28	5.90	3.19	2.44
	0.12	4.64	3.40	2.09	1.65
	0.15	3.08	2.24	1.62	1.34
	0.18	2.21	1.69	1.33	1.16
	0.21	1.71	1.38	1.19	1.07
	0.24	1.44	1.20	1.10	1.04
	0.27	1.25	1.11	1.04	1.01
	0.3	1.15	1.05	1.02	1.01
50	0	68.93	69.32	48.43	40.50
	0.03	19.58	15.89	8.69	6.95
	0.06	8.04	6.18	3.18	2.64
	0.09	3.93	2.84	1.83	1.59
	0.12	2.34	1.79	1.34	1.21
	0.15	1.65	1.36	1.15	1.09
	0.18	1.30	1.16	1.06	1.03
	0.21	1.15	1.07	1.02	1.01
	0.24	1.07	1.03	1.01	1.00
	0.27	1.02	1.01	1.00	1.00
	0.3	1.01	1.00	1.00	1.00
70	0	75.98	65.67	45.59	40.19
	0.03	16.42	13.83	8.01	6.06
	0.06	6.07	4.63	2.75	2.19
	0.09	2.89	2.17	1.57	1.31
	0.12	1.76	1.40	1.20	1.10
	0.15	1.33	1.14	1.06	1.03
	0.18	1.14	1.06	1.02	1.01
	0.21	1.04	1.02	1.00	1.00
	0.24	1.01	1.00	1.00	1.00
	0.27	1.00	1.00	1.00	1.00
	0.3	1.00	1.00	1.00	1.00

4. Conclusions

A new multiatributte double sampling control chart has been proposed. The simulation results suggest that the proposed control chart is more efficient than the D^2 control chart in almost all simulated cases. The study also suggest, as expected, that using double sampling size improves the performance of control charts without increases the average number of inspected items. The performance of both control charts are strongly affected for the decreasing of sample size. The effect is bigger if the shift in the defective proportions decreases. As expected, the number of categories constrains the sample size, this is, *n* must be longer than *k* to assurance detection efficiency of special causes of variation. Obtain optimal designs for the proposed control chart; propose a method to identify the defect that causes an out of control signal and improve control chart sensitivity to detect small shifts is left as future work.

5. References

Costa, A.F.B. and Machado, M.A.G. (2011). Variable parameter and double sampling X charts in the presence of correlation: The Markov chain approach. *Int. J. Production Economics*, 2011; 130: 224–229.

Daudin, J.J. (1992). Double Sampling X Charts. *Journal of Quality Technology*. 24 (2): 78–87.

De Magalhães, Maysa S., Costa, A. F. B. and Moura, F. D. N. (2009). "A hierarchy of adaptive $\overline{X}$ control charts. *Int. J. Production Economics*, 2009; 119: 271–283.

Faraz, A., Kazemzadeh R., Parsian A. and Moghadam M. B. (2012). On the advantages of economically designed the Hotelling's T2 control chart with variable sample sizes and sampling intervals. *Qual Quant*, 2012; 46:39–53.

Ghute, V. B. and Shirke, D.T. (2008). A multivariate synthetic control chart for process dispersion. *Quality Technology & Quantitative Management*, 2008; 5(3): 271-288.

Haridy, S., Wu, Z., Abhary, K., Castagliola, P., and Shamsuzzaman, M. (2013). Development of a multiattribute synthetic-np chart, *Journal of Statistical Computation and Simulation*, 2013; DOI:10.1080/00949655.2013.769541.

He, D., Grigoryan, A. and Sigh, M. (2002). Design of double- and triple-sampling X-bar control charts using genetic algorithms. *International Journal of Production Research*, 2002; 40 (6): 1387-1404.

Irianto, D. and Juliani, A. (2010). A two control limits double sampling control chart by optimizing producer and customer risks. *ITB J. Eng. Sci.*, 2010; 42(2):165-178.

Jolayemi J.K. (1999). A statistical model for the design of multiattribute control charts. *Indian J Stat*. 1999; 61: 351–365.

Khoo, M. B. C., Lee H. C., Wu Z., Chen C. and Castagliola P. (2010). A synthetic double sampling control chart for the process mean. *IIE Transactions.* 2010; 43(1): 23-38.

Khoo, M.B.C., Wu Z., Castagliola, P. and Lee, H.C. (2013). A multivariate synthetic double sampling T^2 control chart. *Computers & Industrial Engineering*, 2013; 64: 179–189.

Lu, X.S., Xie M., Goh T.N., and Lai, C.D. (1998). Control chart for multivariate attribute processes. *Int J Prod Res*, 1998; 36: 3477–3489.

Mahadik, S. B. and Shirke, D. T. (2011). A special variable sample size and sampling interval Hotelling' s T^2 Chart. *Int J Adv Manuf Technol*, 2011; 53: 379 – 384.

Mukhopadhyay, A.R. (2008) Multivariate attribute control chart using Mahalanobis D^2 statistic. J Appl Stat. 35: 421–429.

Niaki, S. T. A. and Nasaji, S. A. (2011). A hybrid method of artificial neural networks and simulated annealing in monitoring auto-correlated multi-attribute processes. *Int J Adv Manuf Technol*, 2011; 56:777–788.

Seif, A., Moghadam, M. B., Faraz, A. and Heuchenne, C. (2011). Statistical merits and economic evaluation of t^2 control charts with the vssc scheme. *Arab J Sci Eng*. 36:1461–1470.

Taleb H. and Limam, M. (2006). Multivariate fuzzy multinomial control charts, *Quality Technology & Quantitative Management*, 2006; 3(4): 437-453.