Systematic Routine for Setting Confidence Levels for Mean Time to Failure (MTTF)

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Abstract: There are circumstances where an item is intentionally tested to destruction. The purpose of this pedagogy is to determine the failure rate, λ, of a tested item. For these items, the quality attribute is defined as how long the item will last until failure. Once the failure rate is determined from the number of survivors and total time of all items tested, the mean time to failure (MTTF), which is a typical statistic for survival data analysis issues, is calculated by dividing one by the failure rate, λ. From this calculation one obtains the reliability function $R(t) = e^{-\lambda t}$ where t is time. This allows the cumulative density function $F(t) = 1- e^{-\lambda t}$ to be determined. This density function, $f(t) = \lambda e^{-\lambda t}$ is a negative exponential with a standard deviation of $1/\lambda$. Thus setting a warranty policy for the tested item is difficult for the practitioner. An important property of the exponential distribution is that it is memory less. This means its conditional probability follows $P(T > s + t \mid T > s)=P(T > t)$ for all $s,t \geq 0$. The exponential distribution can be used to describe the interval lengths between any two consecutive arrival times in a homogeneous Poisson process. The purpose of this research paper is to present a simple technique (pedagogy) to determine a realistic warranty level for a tested item can be predicted. Simple failure estimates for establishing a warranty pay-out cannot be calculated without a standard deviation. This procedure will allow one to set a warranty policy that will allow a very small percent of failures to be honored. For example let us wish to only honor 1% of the failures. Then 99% will not be honored. Thus this pedagogy is most practical and useful for making an initial estimate.

Keywords: Mean Time to Failure, MTTF, negative exponential distribution, Kaplan-Meier estimator

1. Introduction

There are circumstances where an item (system) is intentionally tested to destruction. The purpose of this technique is to determine the failure rate, λ, of a tested item. Its principle use is to assist in projecting the probabilities over time before a system would fail. As such failures become a function of time (Slack, 2001). Such analysis results have long been used to assist in describing optimal maintenance periods, warranties, or useful life (Grosh, 1989 and Juran 1988).

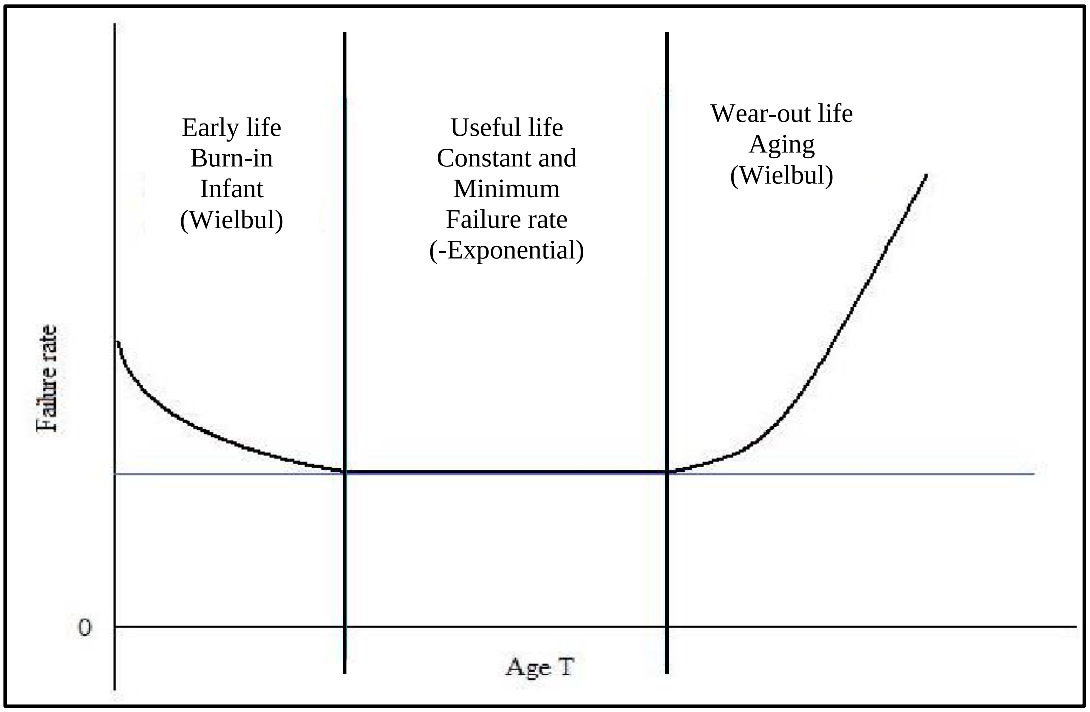

Figure 1. Traditional Bathtub Curve (Grosh, 1989)

In its initial application survival data analysis used the bathtub curve was used to describe the application of this phenomenon (Jenson, 1990). The survival and failure curve was divided into three parts as illustrated in Figure 1. Imperial data was determined to be exponentially distributed. Early life, burn-in, or infant failures were determined to be Weibull distributed ($0<\beta<1$). Useful life, constant or minimum failure was determined to be exponentially distributed. Wear-out life or Aging was determined to be Weibull distributed ($\beta>1$) (Grosh, 1989). Over the years the bath-tub curve especially the burn-in and wear-out periods has been challenged for its product usefulness (Grosh, 1989 and George, 2000). This was especially true in accelerated testing. For useful life the exponential distribution appears to be quite appropriate (Grosh, 1998). Once the failure rate is determined from the number of survivors and total time of all items tested, the mean time to failure (MTTF) which is a typical statistic for survival data analysis issues can be calculated. In the following discussion and derivation one can observe that the standard deviation becomes the MTTF. Therefore more complex methods are required to determine the confidence level of failure. Many practitioners are not as familiar with the appropriate techniques to obtain a simpler solution to this problem. The purpose of this research paper is to present a simple technique to determine a realistic confidence level. Using the same technique the warranty level for the tested system can be predicted.

2. Mean Time to Failure

MTTF is calculated by dividing one by failure rate, λ. From this one obtains the reliability function $R(t) = e^{-\lambda t}$ where t is time. This allows the cumulative density function $F(t) = 1- e^{-\lambda t}$ to be determined. This density function, $f(t) = \lambda e^{-\lambda t}$ is a negative exponential with a standard deviation, σ, = $1/\lambda$. Thus, with the standard deviate equal to MTTF, setting a warranty policy for the tested item is difficult for the practitioner. An important property of the exponential distribution is that it is memory less. This means its conditional probability follows $P(T > s + t \mid T > s)=P(T > t)$ for all $s, t \geq 0$. The exponential distribution can be used to describe the interval lengths between any two consecutive arrival times in a homogeneous Poisson process.

2.1 Defining the Derivation

Before deriving the MTTF the following parameters and definitions will be required. Figure 2 is used to assist in describing these parameters.

The reliability function $P(T>t) = R(t) = e^{-\lambda t}$. Where $0 \le t < \infty$

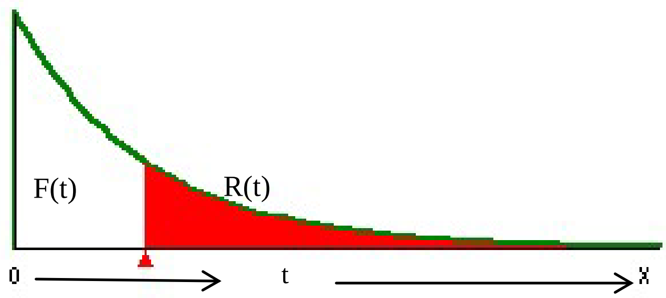

Figure 2. Exponential Distribution (Grosh, 1989)

The cumulative density function $P(T < t) = F(t) = 1-R(t) = 1-e^{-\lambda t}$

The probability density function $f(t) = \lambda e^{-\lambda t}$

From these parameters we can conclude that the failure rate then becomes λ = (# failures)/(Σ survival time) Thus equation 1 is the survival rate (Equation 1).

$$\text{Failure Rate} = \lambda \quad (1)$$

2.2 Derivation

Thus the Mean Time To Failure (MTTF) is derived as follows (Grosh, 1989):

$$\text{MTTF} = \int_0^\infty e^{-\lambda t} dt$$

$$\text{MTTF} = -\frac{1}{\lambda} e^{-\lambda t} \Big|_0^\infty$$

$$\text{MTTF} = (-1/\lambda e^{-\lambda(\infty)}) - (-1/\lambda e^{-\lambda(0)})$$

$$\text{MTTF} = 0 - (-\frac{1}{\lambda}) = \frac{1}{\lambda}$$

Thus the mean time to failure time is determined by equation (2)

$$\text{MTTF} = 1/\lambda \quad (2)$$

2.3 Example

As a simple example 12 items will be tested to determine their MTTF. It was determined prior to the testing that systems would only be tested to 13,000 hours. Two of the 12 items will survive the test. Table 1 and Figure 3 illustrate the results.

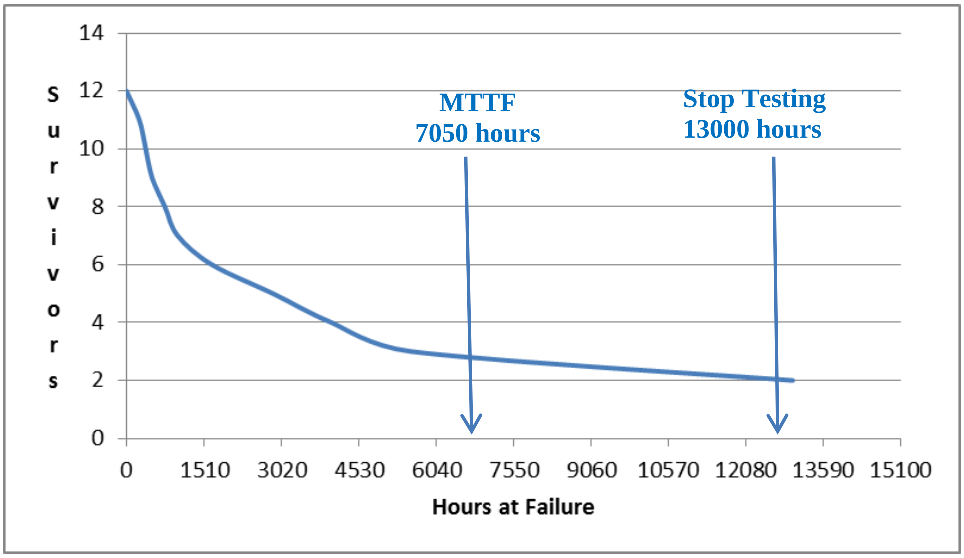

Figure 3. MTTF Results

λ = number of failures/total time = 10/70,500 = 0.000141844

MTTF = 1/ λ = 1/0.000141844=7050 hours

Table 1. Data for MTTF Example

Failures	Hours at Failure
1	0
2	250
3	500
4	750
5	1500
6	2000
7	5000
8	10000
9	12000
10	12500
11	13000
12	13000

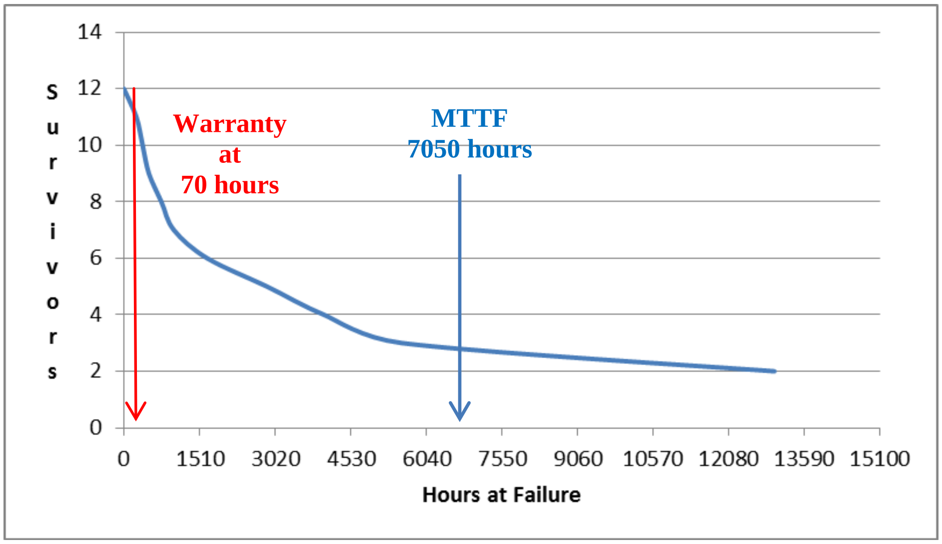

Figure 4. Warranty Estimate

2.4 Setting the Warranty

Now that that the the MTTF of 7050 hours has beeen calculated, the item's warranty can be calculated. For the example in paragraph 2.3, the desired warrenty payout is to only pay for 1% of the early constant failures. As stated earlier , the standard deviation is also equal to 1/λ or 7050 hours. Although a rather complex integral to be can be used for this calculation, this method produces a good estimated time to set a warrenty or to assist in developing preventative maintenance tables. The desired limit for warranty payout will be 1% or a relability of 99%. To find the failure time in the lowest 1%, the following is based upon the basic principles forund in Paragraph 1 and 2 and is as follows:

$$P(T < t) = F(t) = 1\text{-}R(t) = 1\text{-}e^{-0.000141844t} = 0.01$$

$$e^{-0.000141844t} = 0.99$$

$$-0.000141844t = \ln(.99)$$

$$t = (-\ 1/\ 0.000141844)*(\ln(.99)) = 70.85 \text{ hours}$$

Therefore, the warranty would set at 70 hours. Note that to safeside this estimate, round down to a number that can be remembered (Figure 4).

3. Setting the Warranty

3.1 Censoring

Censoring occurs for different reasons and in different forms. We will especially focus on a right censoring. We say that an observation on a variable T is right censored if all one can know about T is that it occurs after some value T*. Right censored survival data are very common and need special treatment. The most common situation of right censored data is depicted in Figure 5. Suppose that this figure depicts 5 items. The horizontal axis represents time. Each of horizontal lines represents a different item. The vertical line at 3 is the time when one stops observing the items. Any failure occurring at time 3 or earlier is observed. Random censoring can also be occurred when there is a single termination time like Item 4 in Figure 5, but entry time varies randomly across individuals.

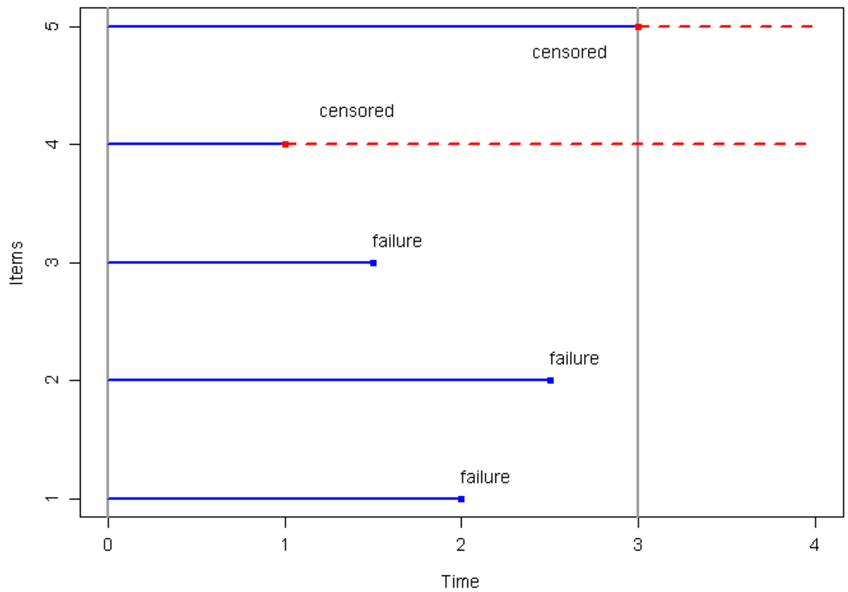

Figure 5. Right-Censored Data

When variables are continuous, another common way of describing the probability distribution is the hazard function, the instantaneous risk that an event will occur at time t. The hazard function is defined as $h(t) = \lim_{s \to \infty} \frac{P(t \le T < s+t | T \ge t)}{s}$. It is not really a probability as the hazard can be greater than 1.0. Furthermore, the hazard has no upper bound but cannot be less than zero. Figure 6 illustrate the relationship between the survival function and the hazard function for Weibull distribution with different parameters.

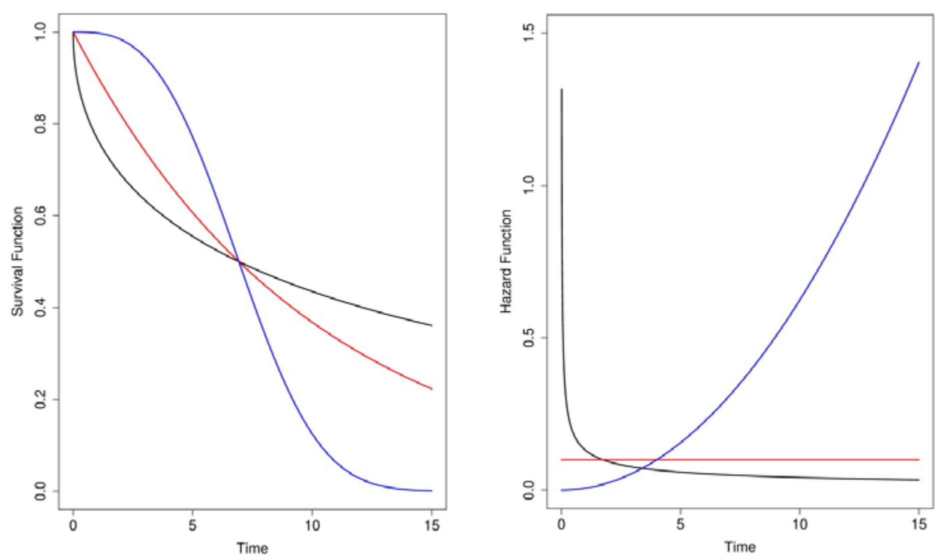

Figure 6. Survival Function vs Hazard Function for:
β = 0.5, λ = 0.26328 (black), β = 1.0, λ = 0.1 (red), and β = 3.0, λ = 0.00208 (blue)

In various areas, the Kaplan-Meier estimator is the most widely used method for estimating survivor functions. This estimator is also known as the product-limit estimator. The Kaplan-Meier estimator, $\hat{S}(t)$, is the proportion of observations in the sample with failure times greater than t when there is no censoring. To be more realistic, one can consider that some failure times are censored. Suppose there are k distinct failure times, $t_1 \le t_2 \le \cdots \le t_k$. Let d_j be the number of objects who fail at time t_j and n_j be the total number of objects who survived before time t_j. The Kaplan-Meier estimator is defined as $\hat{S}(t) = \prod_{j:t_j \le t}(1 - \frac{d_j}{n_j})$ for $t_1 \le t \le t_k$. This estimator takes all the failure times that are less than or equal to t and computes an estimate of the conditional probability of surviving to time t_{j+1}. Then all these probabilities would be multiplied together.

3.2 Examples and Setting the Warranty

To illustrate our approach, an example was considered. For example, consider 200 items with the survival function, $R(t) = e^{-1/7050t}$ and the censoring function, $C(t) = e^{-1/18000t}$. The parameter of censoring function was carefully chosen so that approximately 30% of failure times would be censored. Figure 7 illustrates the results.

An estimated MTTF can be calculated directly from the estimated survival function. As shown in Figure 7, the mean is biased downward when there are censoring times greater than the largest failure time. It is because about 30% of failure times are censored and the survival probability was estimated based on the information that the failure occurred after the censoring time. Also note that the survival probability does not reach zero with censoring. If the limit for warranty payout is 1% or a reliability of 99%, the warranty would set at 70 hours as the 1st percentile of the observed time is 70.77 hours in the simulation using bootstrapping method.

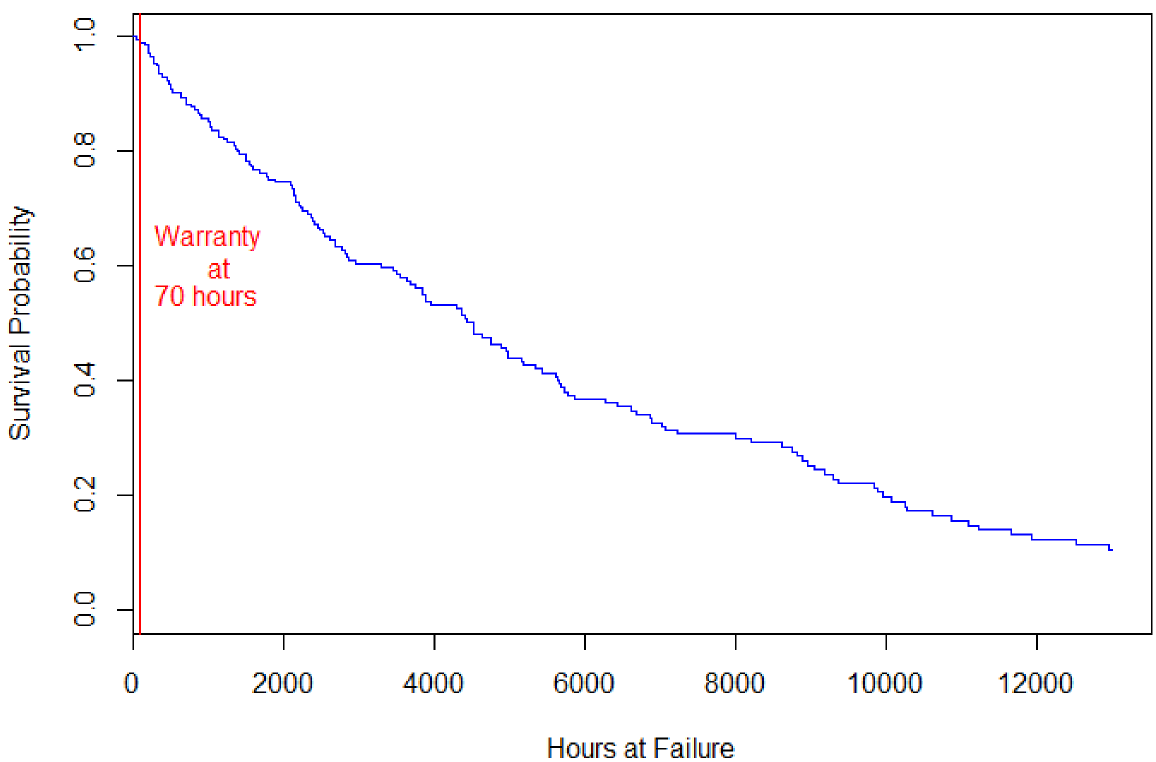

Figure 7. Kaplan-Meier Survival Estimate

4. Application and Conclusion

This procedure presents a very simple technique to determine a realistic confidence level for setting a simple system or product's warrant for early failure. Also it may be applied to relatively unsophisticated survival data analysis. A 99% probability for reliability is not unrealistic in today's competitive marketplace. Also this technique is useful in introducing reliability theory to both undergraduate and graduate students. Of course the more serious students will pursue much more rigorous experimental design and statistical procedures.

5. References

Allison, P. (2010), *Survival Analysis Using SAS*, 2nd Edition, SAS Institute Inc, Cary.

George, L (2000, February). The Bathtub Curve Doesn't always Hold Water. Retrieved from http://www.asq-rd.org/articleBathtub.htm.

Grosh, D. L. (1989). *A primer of reliability theory*. New York: Wiley.

Jensen, F (1990), *Burn-in: An Engineering Approach to the Design and Analysis of Burn-in Procedures*, 7th Edition, John Wiley & Sons, Chichester

Juran, J. M (2010), *Quality Control Handbook*, 6th Edition, McGraw-Hill, New York.

Klein, J. P., Moeschberger, M. L. (2005), *Survival Analysis: Techniques for Censored and Truncated Data*, 2nd Edition, Springer, New York

Slack, N. (2001), *Operations Management*, 3rd Edition, Prentice Hall, London.