The Mathematical Underpinnings of Defensive Counterair Operations in Great Power Competition

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Abstract: In the context of the Great Power competition, particularly with China's advancing long-range munitions capabilities, there is a critical need to adapt defensive counterair (DCA) operations planning. This paper addresses some challenges of the complex and time-consuming DCA planning processes, often resulting in suboptimal decisions. We explore the mathematical foundations of DCA operations and provide a closed-form solution that determines aircraft requirements based on desired operational distances and calculates required distances based on aircraft availability. This approach helps to enable the Agile Combat Employment (ACE) concept, offering increased flexibility in response to potential denial of access to predetermined locations. We demonstrate the applicability of our equations through a couple of notional examples and simulations, showing potential for more than 75%-time savings improvement for the calculations compared to current methods and improved mission effectiveness from improved aircraft allocation techniques. This potential increase in efficiency and effectiveness could improve the ability to adapt to changing strategic environments, supporting the ACE framework's goals of increased survivability and operational flexibility in contested spaces.

Keywords: Agile Combat Employment, Combat Air Patrols, Defensive Counterair, Missile Defense, Operations Planning, Simulation

1. Introduction

In the evolving landscape of Great Power competition (GPC), the United States Air Force faces unprecedented challenges in maintaining air superiority and protecting its assets. As the People's Republic of China (PRC) continues to develop its military capabilities at an alarming rate, the need for adaptive and resilient defense strategies has never been more critical. The Air Force's response to this growing threat is the concept of Agile Combat Employment (ACE), a modernized power-projection approach designed to enhance survivability and operational flexibility in contested environments.

Just as it was historically proven in the battle of Agincourt that stopping the archer is much easier than stopping their arrows once launched, today's missile defense is the same (Sumption 2017). The PRC's expanding missile arsenal and anti-access/area-denial (A2/AD) capabilities pose a significant threat to traditional U.S. air operations, particularly in the Pacific theater. These advancements have forced the Air Force to transition from large, centralized bases to smaller, dispersed, and resilient adaptive basing strategies.

The challenge of modern Defensive Counterair (DCA) operations planning in the context of ACE is multifaceted, requiring simultaneous solutions to three intricate and interrelated problems:

1. Determining the optimal number and type of aircraft required to minimize threats in a dispersed environment.
2. Maintaining logistical supply chains for sustained operations across multiple locations for an unknown period.
3. Optimizing asset deployment for both homeland and unit protection while maintaining unpredictability.

Answering any of these three problems to optimality is difficult at best, and the combination of all three of them together presents a computationally intractable scenario considering all the unknowns of the threat assessment, mission objectives, desired operations tempo, adversary capabilities, and resource availability. In addition, a mission planner understands that solving these large and important planning scenarios requires trade-offs between the objectives. For example, the number of aircraft available limits the number of aircraft needed for a DCA mission, while current doctrine often shapes the employment of available assets. Further complications arise from the PRC's ability to potentially deny access to

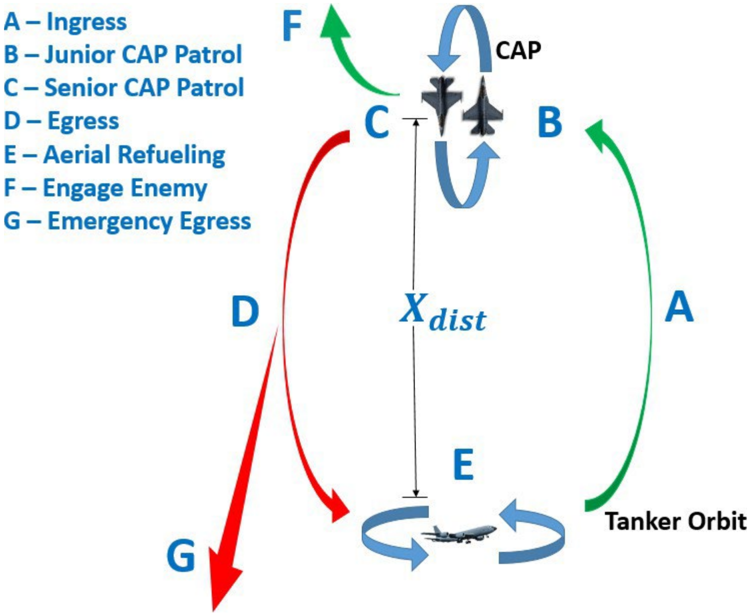

Figure 1. Combat Air Patrol Racetrack Diagram

predetermined ACE locations through diplomatic, economic, or kinetic action. As such, the Air Force must develop flexible, scalable, and rapidly adaptable strategies for DCA operations that can respond to dynamic threat environments.

This paper aims to address parts of the first and third problems by developing mathematical models that can inform DCA mission planning and determine ACE site selection. By providing closed-form solutions to aircraft and distance requirements, we seek to streamline the planning process and enable rapid decision-making capabilities for adaptive basing. Before we explain the solution methodology, it is important to understand the current DCA operations and planning procedures.

1.1 Current DCA Operations

In a typical DCA operation today, defensive aircraft often fly in pairs some distance ahead of the tanker orbit. This two-ship patrol formation, vital for mission safety and success, allows friendly forces to have continuous protection from enemy threats (i.e., coverage). In this paper, we define this grouping more generically as patrols which can take on any number of *n*-ship deployments. Figure 1 shows a typical progression of DCA aircraft through the combat air patrol (CAP).

The cyclical DCA operational pattern will be referred to as the racetrack. After a patrol refuels, it progresses to position A where it transitions some distance, X_{dist}, to the CAP location *B*. Traditionally X_{dist} is relatively small, however improving technology is changing the face of traditional DCA. Due to the increased risk of enemy technology, forward positioning CAPs are required to keep tankers safe. If the CAP requires two patrols to be fully manned, they will then transfer to position *C* where the patrol will continue defensive posturing in the CAP. While in the CAP, the patrols have the option to: 1) Push forward and defend against enemy incursion shown in position *F* or; 2) Transition back to the tanker shown by position *D* when a replacement patrol arrives. The patrol then reconnects with the tanker to refuel and continue the cycle. Patrols running this racetrack must consider emergencies that may require them to divert to a safe location. The minimum amount of fuel each aircraft is required to maintain as a safety buffer is known as bingo fuel. Bingo fuel includes the predetermined fuel reserve plus the amount of fuel required to return to base. This emergency fuel requirement must be considered when determining the possibility of diverting to a safe location as shown in position *G*. Improving technology as referenced by Foster (2018) necessitates a deviation from traditional DCA tactics. One way to mitigate the threat of long-range enemy munitions is by increasing the distance we position the CAPs ahead of the tanker orbit. Doing this, however, requires more complicated planning and logistical considerations than have been required in the past. The paper addresses these complexities of DCA operations by mathematically analyzing and providing a closed-form solution to aircraft and distance requirements, aiming to streamline and expedite parts of the planning process for DCA.

For example, a planner desires to determine how many fighter patrols are needed to conduct DCA in a threat region. By setting requirements on the CAP coverage and enemy threat distances, it is possible to quickly solve for the optimal number of patrols needed with the closed form solution that will be described in this paper. This will greatly help the mission planners set this high-level resource allocation and plan for all the support that is needed. Additionally, other factors can be held constant and different objectives optimized. One example is instead of solving the optimal number of patrols, the patrols are set to the number of aircraft available. Then other factors can be optimized such as the maximum distance that the CAP should be flown from the tankers to achieve DCA coverage requirements. There are many other possible objectives and uses, but in just these two examples, the number of iterations of the planning process that can be achieved in a fraction of the current planning time in addition to the number of tradeoffs discussed and analyzed will be multiplied. The overall impact of utilizing these mathematical models and solutions will greatly improve mission effectiveness. Next, we will discuss how and when these calculations can be applied in the current Joint Air Operations Planning process.

1.2 Current Joint Air Operations Planning

Joint air operations planning is crucial for ensuring the effective integration and coordination of airpower within military operations. This planning and operations process enhances the operational efficiency by aligning the efforts of all branches of the armed forces. Planning typically follows a structured approach, often encapsulated in the Joint Planning Process for Air (JPPA), which includes steps such as mission analysis, course of action development, and execution planning adaptive the joint planning process from Joint Publication 5-0, Joint Planning. The JPPA includes steps such as mission analysis, course of action (COA) development, and execution planning. By systematically assessing the operational environment and determining the best use of air assets, commanders can optimize their strategies to achieve mission objectives while adapting to dynamic battlefield conditions.

According to Joint Publication 3-30 from the Joint Chiefs of Staff (2019), joint air operations planning begins with the Joint Force Commander's (JFC) mission and intent. The JFC's assessment of the operational environment and objectives guides the components' goals. The Joint Force Air Component Commander (JFACC) utilizes this information, along with the concept of operations (CONOPS), to develop a COA. Once approved by the JFC, this COA forms the foundation for detailed air operations planning, delineating the anticipated impact of air operations. The JFACC's daily guidance ensures that air operations support joint force objectives while maintaining the flexibility to adapt to evolving military situations (as illustrated in Figure 2).

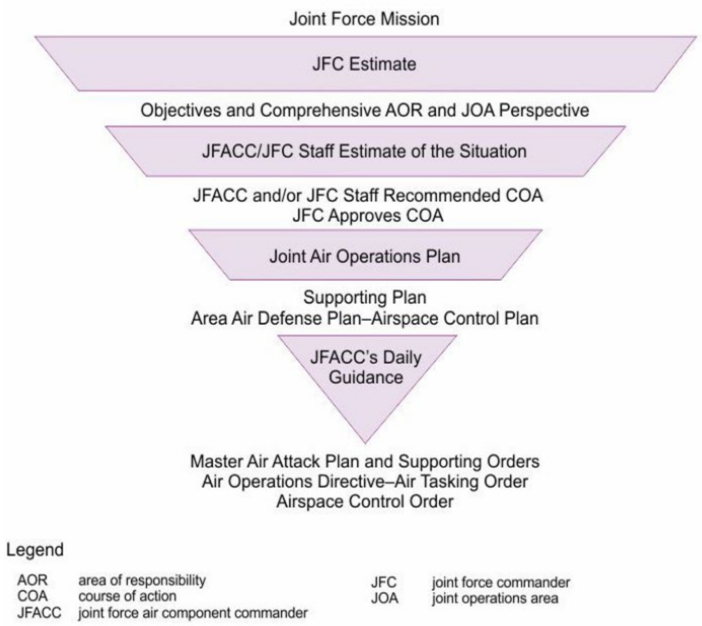

Figure 2. Joint Air Operations Planning, Source: Joint Chiefs of Staff (2019) JP 3-30

The joint air tasking cycle aligns with the JFC's objectives and battle rhythm in the overall Joint Air Operations Planning process. It produces the Air Tasking Order (ATO), which outlines joint air operations for a 24-hour period. The

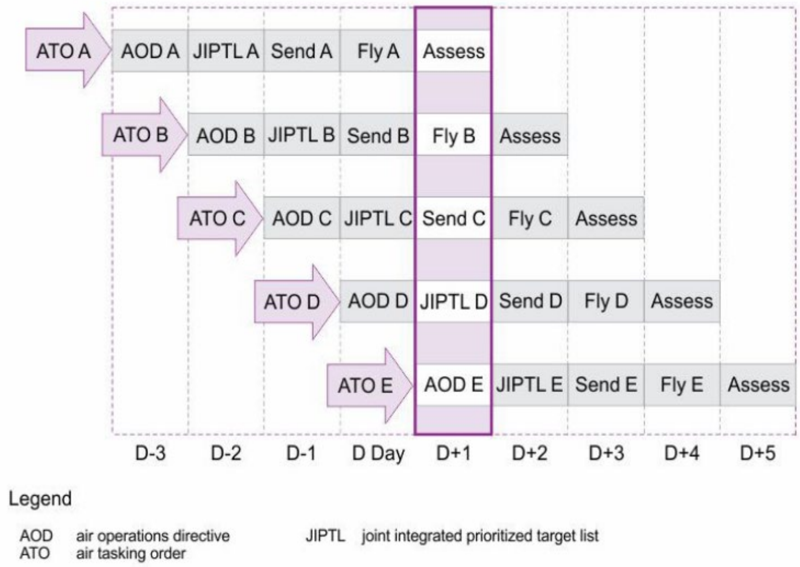

Figure 3. JAOC ATO Schedule Development, Source: Joint Chiefs of Staff (2019) JP 3-30

Joint Air Operations Center (JAOC) typically uses a 72- to 96-hour planning cycle. Two main groups are responsible for constructing the plans: the strategy division, and the combat plans division. The strategy division concentrates on long-range to near-term planning of joint air operations to meet the JFC's objectives and focuses on the resource allocation and assessment beyond the 48-hour prior to ATO execution. The combat plans division is responsible for near-term air operations planning within 48 hours prior to ATO execution and focuses on the execution details of a mission. The daily operations schedule coordinates briefings, meetings, and reporting requirements to create the Air Battle Plan (ABP), which includes the ATO. This process ensures synchronization with the JFC's CONOPS and other components' activities. While structured, the cycle must remain flexible to adapt to dynamic operational environments. At any given time, multiple ATOs are in various stages of development (see Figure 3). The operations schedule drives the ATO development, facilitating timely decision-making through a series of coordinated events.

The joint air tasking cycle is a six-stage process that aligns air operations with the JFC's objectives (see Figure 4). It begins with setting objectives and guidance, including the JFC's air apportionment decision. The cycle then progresses through target development, weaponeering and allocation, ATO production and dissemination, execution planning and force execution, and finally, assessment. The process involves close collaboration between various components and centers, such as the JAOC and the Joint Targeting Coordination Board (JTCB). The cycle is designed to be flexible, allowing for adjustments during execution to respond to changing priorities or emerging targets. Assessment is conducted at both tactical and operational levels, providing feedback for future planning and informing the JFC's overall assessment of the operation's success.

1.3 DCA Specifics in Joint Air Operations Planning

The Joint Air Operations Process applies to all Air Operations, but this paper focuses specifically on DCA operations and planning. The key aspects of DCA fall under Active Air Defense (AAD) planning from Joint Publication 3-01: *Countering air and Missile Threats* (2024), as one element of the Joint Air Operations planning process. One of the key components of the AAD is determining the Weapon Engagement Zones (WEZs). These are specific areas of airspace where particular weapon systems are responsible for engaging threats. One type of WEZ that is important for this paper is the Fighter Engagement Zone (FEZ). This can be viewed as a designated playground for fighter jets, usually high up and far out, beyond the reach of ground-based defenses. In these zones, aircraft can quickly respond to enemy air attacks, no matter where they come from.

In addition, another key source of output from the AAD plan are the CAPs, as described previously. These are the groups of two to four fighter jets armed and ready for air-to-air combat. CAPs can be used in various ways – they can protect broad areas or guard specific high-value assets. Our paper engages in determining both of these critical outputs, that would typically be calculated inside the 48-hour window by the combat plans division. In addition, our paper engages with the strategy division team and the resource allocation problem in determining how many aircraft are needed to meet the JFC objectives, which will ultimately determine the FEZ and CAP outputs.

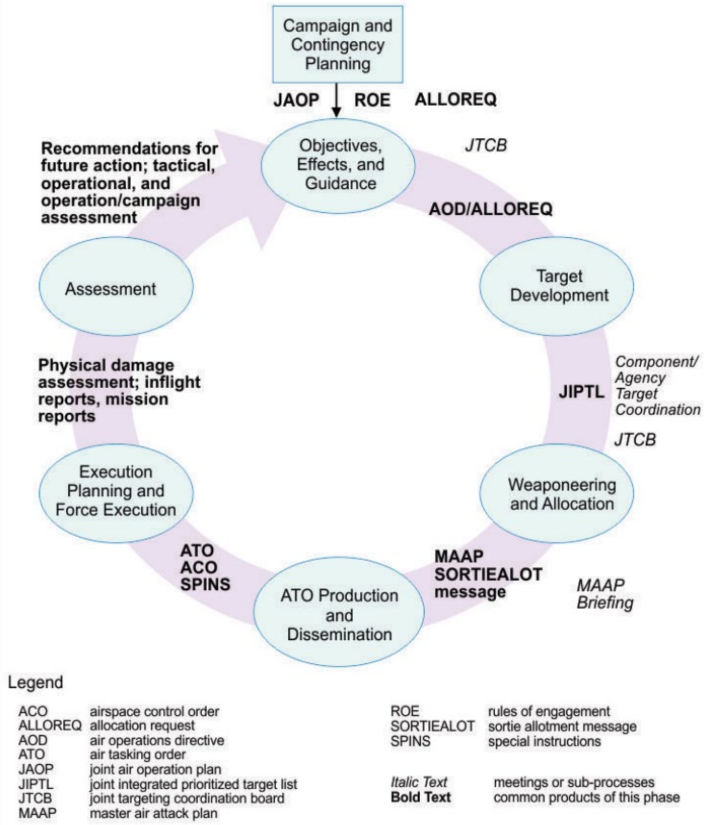

Figure 4. Joint Air Tasking Cycle, Source: Joint Chiefs of Staff (2019) JP 3-30

1.4 Organization

As we delve into the complexities of modern DCA operations, it is crucial to understand how the traditional combat air patrol (CAP) model must evolve to meet the demands of Great Power competition. The following sections will explore the mathematical foundations for optimizing DCA in an ACE framework, providing planners with tools to enhance mission effectiveness and resilience in the face of advanced peer adversary threats. Section 2 – Literature, reviews relevant work on DCA planning problems research, Section 3 – Results and Analysis, addresses our methodological approach, Section 4 – Computational Experiments, gives a small notional example to demonstrate the utility of the methodology and explores a more realistic scenario through simulation, and the final Section 5 – Conclusions and Future Research, summarizes the conclusions, recommendations, and avenues for further research.

2. Literature Review

The Curtis E. Lemay Center for Doctrine Development and Education define counterair operations as the integration of offensive and defensive operations to attain and maintain a desired degree of control of the air and protection by neutralizing or destroying enemy aircraft and missiles, along with threats to air operations from other domains (USAF 2023). The goal of counterair operations then, is to achieve as much control of the air domain as possible. This level of control can be described on a continuum ranging from air parity (i.e., no force has control of the air), air superiority (i.e., a degree of control that permits operations without interference for a given amount of time), and air supremacy (i.e., opposing force incapable of effective interference).

Combat air patrols (CAPs) are an integral part of missile defense. CAPs include both offensive and defensive counterair (DCA) operations that are an integral part of homeland and global missile defense. DCA operations include both active and passive operations: active DCA operations include direct defensive action taken to destroy, nullify, or reduce the effectiveness of hostile air and ballistic missile threats against friendly forces and assets; passive DCA operations include all measures other than active operations to include but not limited to camouflage, concealment, deception, hardening, dispersion, redundancy, and mobility. Active air defense includes defensive measures to destroy attacking aircraft and aerodynamic missiles, or to nullify or reduce the effectiveness of such an attack. It includes the use of aircraft, surface-to-air missiles, antiaircraft artillery, electromagnetic warfare, multiple sensors, and other available weapons or capabilities (USAF 2022).

2.1 Defensive Counterair Operations Problems

Much of the existing research on DCA has focused on the surface-to-missile (SAM) component, particularly the weapon-target assignment problem (WTAP) and the optimal location of SAM batteries problem. The WTAP seeks to deploy a fixed amount of offensive weapons to inflict the maximum damage to a fixed number of targets. Lloyd and Witsenhausen (1986) proved that the general WTAP is NP-complete, making it computationally challenging for non-trivial scenarios. Researchers have proposed various approaches to address this complexity. Brown et al. (2005) developed quick, albeit non-optimal, solutions for naval defensive asset prepositioning. Ahuja et al. (2007) proposed exact algorithms for smaller instances and heuristic algorithms for larger scenarios. Davis, Robbins, and Lunday (2017) addressed a dynamic variant of the WTAP using approximate dynamic programming.

For the optimal location of SAM batteries problem, game theory approaches have also been applied to this type of DCA planning. Game theory is utilized because the defender in the problem must anticipate the attacker's best action to inform their ultimate decision. This type of bilevel optimization comes from what is known as a Stackelberg game (Von Stackelberg 2010). Boardman, Lunday, and Robbins (2017) used game theory to determine optimal SAM battery locations. Tsamtsaridis (2011) employed a defender-attacker model using network interdiction to account for strategy adjustments in DCA scenarios.

Haywood, Lunday, and Robbins (2022) proposed a solution to this type of bilevel optimization via transformations and reformulations to ultimately identify a single mixed-integer nonlinear program. While these studies have significantly contributed to DCA research, they primarily focus on SAM deployment and general resource allocation. Our research differs by specifically addressing airborne DCA planning problems, solving the required number of aircraft and optimal racetrack planning factors, and not the placement of batteries, assignment of weapons to targets, nor how to deploy all assets to defend a set or targets.

2.2 Agile Combat Employment and Site Selection

Agile Combat Employment (ACE) is a U.S. Air Force operational concept designed to counter emerging threats by enhancing force adaptability and resilience in contested environments. ACE centers around adaptive basing, where forces disaggregate from large, centralized bases into smaller, dispersed locations to maintain operational flexibility. This approach leverages both domestic and foreign infrastructure to create a hub-and-spoke network, allowing forces to maneuver and sustain airpower despite adversary actions aimed at undermining access and resources. To address site-selection challenges inherent in ACE—such as geopolitical risks and logistical constraints—multi-criteria decision analysis and geographic information systems (GIS) aid in evaluating potential locations for strategic viability. This methodology supports the Air Force's commitment to maintaining operational readiness in the face of complex threats, particularly from near-peer adversaries like China (USAF 2022).

Recent developments in Great Power competition, particularly with China's rapid military advancements, have necessitated a shift in Air Force strategy. The ACE concept, as discussed by Moer, et al. (2023), represents a critical evolution in power projection and homeland defense. ACE relies on foreign countries' access and infrastructure to generate airpower, complicating traditional DCA planning. Moer et al. (2023) propose an ACE site-selection framework that evaluates basing alternatives using GIS, and analytic hierarchy process to score alternatives. This methodology addresses the challenge of adapting ACE if access to predetermined hubs and spokes becomes compromised due to diplomatic, economic, or kinetic action by adversaries like China. This approach to ACE planning complements our focus on airborne DCA operations by providing a broader strategic context. It highlights the need for flexible, scalable, and rapidly adaptable DCA strategies that can respond to dynamic threat environments and changing basing options.

3. Results and Analysis

To solve the DCA planning problem, we start by simplifying the planning process by mathematically formulating and generalizing defensive counterair (DCA) operations. This provides time saving equations that address two separate planning questions: how many aircraft we need to maintain our desired combat air patrol (CAP), and the maximum distance we can fly ahead of our tanker based on the number of available aircraft. We define k, as the number of patrols needed for continuous coverage. Commonly, DCA requires two patrols in the CAP but we define c more generally as the number of patrols needed in the CAP. Because the flight characteristics are different for each aircraft, we generically define d_i as the maximum fight duration of aircraft patrol i, and x_i as the time patrol i can spend in the CAP (i.e., loiter time). Moreover, we define n_{ij} as the time it takes patrol i to travel any given leg j. Each patrol i, has a defined refueling time r_i. The CAP need not be the same location as the aerial refueling tanker, thus we define the distance from the CAP to the tanker as X_{dist}. The tanker orbit is set up some distance ahead of the origin point; we define this distance from the origin base to the tanker orbit as Y_{dist} as shown in Figure 5. An aircraft can only fly for a fixed duration before returning to their home station. We define this total flight duration for aircraft patrol i to include both travels to and from the racetrack location as T_i. We define the required fuel buffer as required by bingo fuel requirements as B_i. We also define the cruise speed for aircraft i as v_i.

3.1 Number of Combat Air Patrols Calculation

The number of patrols required to fully maintain a CAP is of great importance to DCA planning operations. The key to maintaining full coverage of the DCA operations is the ability to replace patrols in a timely manner. The replacement equation is defined in equation 1.

$$(k-c)x_i \geq c(2n_{i2} + r_i) \tag{1}$$

The replacement equation states that the total amount of time that each patrol spends in the CAP has to be at least as much as it takes to replace them. Total fight time includes both travel to and from the racetrack location as shown in Figure 5. There are two distinct segments of travel for aircraft who fly this mission: 1. n_{i1} is the time to travel to and from home base to the designated tanker orbit (i.e., transit in and egress from a location designated return to base (RTB)); 2. n_{i2} is the time to travel to the CAP and back to the tanker. Moreover, the maximum amount of time an aircraft can fly the racetrack, m, is defined by the total time they can fly the racetrack divided by how long their maximum flight duration is shown in equation 2.

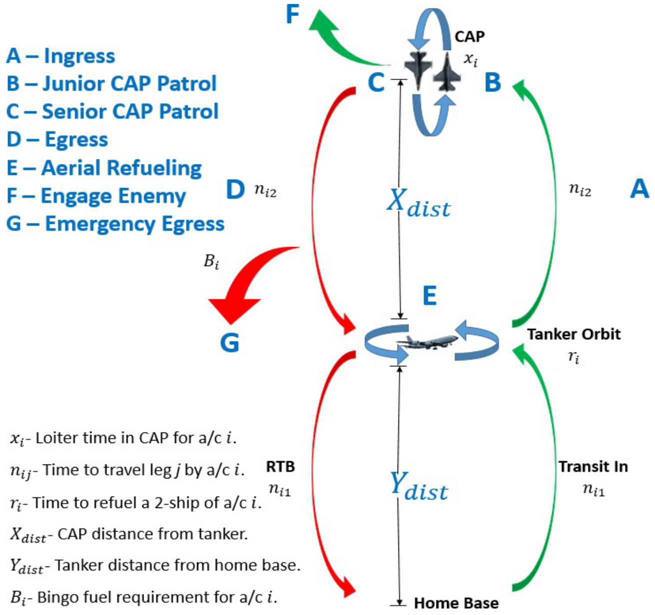

Figure 5. Patrol Path Diagram

$$m = \frac{T_i - 2n_{i1}}{d_i} \tag{2}$$

Total flight time is defined in equation 3.

$$T_i = 2n_{i1} + m(2n_{i2} + r_i + x_i) \tag{3}$$

The number of patrols required to maintain continuous CAP coverage, *k*, can be derived from these three combined equations (i.e., equations 1, 2, and 3). This derivation is shown in equation 4.

$$
\begin{aligned}
(k-c)x_i &\geq c(2n_{i2}+\ r_i) && (4)\\
k &\geq \frac{c(2n_{i2}+r_i)}{x_i}+c\\
T_i &= 2n_{i1}+m(2n_{i2}+r_i+x_i)\\
x_i &= \frac{T_i-2n_{i1}}{m}-(2n_{i2}+r_i)\\
k &\geq \frac{c(2n_{i2}+r_i)}{\frac{T_i-2n_{i1}}{m}-(2n_{i2}+r_i)}+c\\
k &\geq \frac{mc(2n_{i2}+r_i)}{T_i-2n_{i1}-m(2n_{i2}+r_i)}+c\\
m &= \frac{T_i-2n_{i1}}{d_i}\\
k &\geq \frac{\frac{T_i-2n_{i1}}{d_i}c(2n_{i2}+r_i)}{T_i-2n_{i1}-\frac{T_i-2n_{i1}}{d_i}(2n_{i2}+r_i)}+c\\
k &\geq \frac{\frac{T_i-2n_{i1}}{d_i}c(2n_{i2}+r_i)}{T_i-2n_{i1}\left(1-\frac{2n_{i2}+r_i}{d_i}\right)}+c\\
k &\geq \frac{c(2n_{i2}+r_i)}{x_i}+c && (5)
\end{aligned}
$$

Thus, the number of patrols needed to maintain continuous CAP coverage can be discretely defined given the flight characteristics of a given aircraft. This coverage in Equation 5 is particularly useful for DCA planning when you have a set distance ahead of the tanker orbit that you need your CAP to protect (i.e., a fixed X_{dist}). A fixed number of available aircraft, however, may necessarily dictate how many aircraft you can deploy. In cases where aircraft availability is limited, it may be necessary to know how far ahead of the tanker orbit you are able to fly a CAP.

3.2 Maximum Distance for a Combat Air Patrol Calculation

Starting again with the coverage equation, we can define the maximum distance, X_{dist}, you can fly a CAP with a fixed number of available patrols shown in Equation 6 (i.e., fix *k* and solve for X_{dist}). We also note here that in the derivation of the number of patrols needed to maintain full CAP coverage, the distance from the home station to the aerial refueling tanker, Y_{dist}, and the associated time to travel from home station to the racetrack, n_{i1}, is mathematically irrelevant, i.e., as long as the aircraft can enter the racetrack, the distance to the racetrack is extraneous. While this may be true in theory, we note that there are limits on human operators that must be considered that may necessarily eliminate pilots flying continuously for obscenely long durations (e.g., pilots flying 12 hours before arriving at the racetrack to start CAP operations).

$$(k-c)x_i \geq c(2n_{i2}+\ r_i) \tag{6}$$

$$n_{i2} = \frac{X_{dist}}{v_i} \tag{7}$$

$$
\begin{aligned}
(k-c)x_i &\geq c\left(\frac{2X_{dist}}{v_i}+\ r_i\right)\\
X_{dist} &\leq \frac{v_i}{2c}[(k-c)x_i-cr_i]\\
x_i &= d_i-(2n_{i2}+r_i)\\
X_{dist} &\leq \frac{v_i}{2c}[(k-c)(d_i-(2n_{i2}+r_i))\quad -cr_i]\\
X_{dist} &\leq \frac{v_i}{2c}[kd_i-k(2n_{i2}+r_i)-cd_i+c(2n_{i2}+r_i)-cr_i]\\
n_{i2} &= \frac{X_{dist}}{v_i}\\
X_{dist} &\leq \frac{v_i}{2c}\left[kd_i-k\left(\frac{2X_{dist}}{v_i}+r_i\right)-cd_i+\frac{2cX_{dist}}{v_i}\right]\\
X_{dist} &\leq \frac{v_i}{2c}kd_i-\frac{kX_{dist}}{c}-\frac{v_i}{2c}kr_i-.5v_id_i+X_{dist}\\
\frac{kX_{dist}}{c} &\leq \frac{v_i}{2c}kd_i-\frac{v_i}{2c}kr_i-.5v_id_i
\end{aligned}
$$

$$kX_{dist} \le \frac{v_i}{2}kd_i - \frac{v_i}{2}kr_i - \frac{v_i}{2}cd_i$$

$$X_{dist} \le \frac{v_{i(kd_i - kr_i - cd_i)}}{2k} \tag{8}$$

The derivation which results in equation 8 represents a closed-form solution that can efficiently estimate the maximum distance you can fly a CAP ahead of a tanker. Given the number of available aircraft and individual flight characteristics of the selected aircraft, a mission planner can quickly and accurately determine where they need to position their DCA assets to effectively defend against enemy aggression. We will now explore the limit properties of both the coverage and distance equations (i.e., equations 5, 8).

3.3 Proofs for Combat Air Patrol Calculations

THEOREM 1. Assuming a non-zero refuel time, as you shrink the distance between the CAP and the refueling orbit you can never have less than *c* patrols for continuous coverage.

Proof. Starting with the coverage equation 5 and distance definition equation 7, we will show the limit to the number of patrols needed, *k*, shrinks to no less than *c* as the distance from CAP location and tanker orbit shrinks to zero.

$$k \ge \frac{c(2n_{i2} + r_i)}{d_i - (2n_{i2} + r_i)} + c, \; n_{i2} = \frac{X_{dist}}{v_i}$$

$$k \ge \frac{c\left(\frac{2X_{dist}}{v_i} + r_i\right)}{d_i - \left(\frac{2X_{dist}}{v_i} + r_i\right)} + c$$

$$k \ge \frac{c\left(\frac{2X_{dist}}{v_i} + r_i\right)}{d_i - \left(\frac{2X_{dist}}{v_i} + r_i\right)}\frac{v_i}{v_i} + c$$

$$k \ge \frac{2cX_{dist} + cr_iv_i}{d_iv_i - 2X_{dist} - r_iv_i} + c$$

$$k \ge \frac{2cX_{dist}}{d_iv_i - 2X_{dist} - r_iv_i} + \frac{cr_iv_i}{d_iv_i - 2X_{dist} - r_iv_i} + c$$

$$\begin{aligned}
&\lim_{X_{dist}\to +0}\left[\frac{2cX_{dist}}{d_iv_i - 2X_{dist} - r_iv_i} + \frac{cr_iv_i}{d_iv_i - 2X_{dist} - r_iv_i} + c\right] \\
&= \frac{0}{d_iv_i - 0 - r_iv_i} + \frac{cr_iv_i}{d_iv_i - 0 - r_iv_i} + c \\
&= 0 + \frac{cr_i}{d_i - r_i} + c \\
&= \frac{cr_i}{d_i - r_i} + \frac{c(d_i - r_i)}{d_i - r_i} \\
&= \frac{c(r_i + d_i - r_i)}{d_i - r_i} \\
&= \frac{cd_i}{d_i - r_i} \\
k &\ge \frac{cd_i}{d_i - r_i}
\end{aligned}$$

Even with instantaneous refueling time, i.e., $r_i = 0$, the smallest value *k* could take on is *c*.

THEOREM 2. There is a fixed distance limit an aircraft can fly a CAP ahead of a tanker even with unlimited aircraft availability.

Proof. Starting with the distance equation 8, we will show there is a fixed maximum distance as the number of available aircraft becomes unlimited.

$$\lim_{k\to\infty}\frac{v_i(kd_i - kr_i - cd_i)}{2k} = \lim_{k\to\infty}\frac{v_ik(d_i - r_i)}{2k} - \frac{v_icd_i}{2k}$$

$$\lim_{k\to\infty}\frac{v_ik(d_i - r_i)}{2k} - 0 = \lim_{k\to\infty}\frac{v_i(d_i - r_i)}{2} - 0$$

$$\lim_{k\to\infty}\frac{v_i(kd_i - kr_i - cd_i)}{2k} = \frac{v_i(d_i - r_i)}{2}$$

The two theorems show that there is a strict floor to the number of CAPs and that the maximum distance a patrol can fly DCA operations is limited by individual flight characteristics: namely flight speed, maximum flight duration, and refueling time. These results are also quite intuitive; a patrol can go no further forward than half of their maximum flying time (i.e., max duration less time needed to refuel). Understanding this maximum can help DCA planners understand the limits of sending aircraft ahead of the tankers.

4. Computational Experiments

Mission planning and logistical support of defensive operations can be a long and complex task. Planners must account for a plethora of important elements including but not limited to aircraft availability, aerial refueling availability, total coverage time needed, total coverage area, aircraft capabilities, and enemy capabilities. When parsing this large decision space, heuristics must be used to find a solution at the speed of relevance. The above-derived equations can simplify some of the necessary planning assumptions and create actionable plans based on ever-changing inputs.

4.1 Midway Atoll Notional Example

To demonstrate the efficacy of our equations we examine a notional defensive scenario, defending Midway Atoll. To do so, we examine the use of four unique and capable aircraft: F-15C, F-16C, F-18C, and F-22 Block 30/35. The flight capability information we show is obtained via open source, freely accessible, and unclassified information found online. For the purpose of this notional example, the maximum distances used are the ferry distance and the cruise speeds are assumed to be 50% of the maximum speed. Each aircraft's endurance is calculated by the division of the max range by the cruise speed. The collected flight information is summarized in Table 1.

Table 1: Aircraft Performance

Aircraft	Max Range (nmi)	Cruise Speed (kn)	Endurance (hrs)
F-15C	3,000	717	4.19
F-16C	2,200	588	3.74
F-18C	1,500	570	2.63
F-22 30/35	1,800	651	2.76

Table 2: Number of Patrols Needed for Continuous Coverage

Aircraft	Endurance (hrs)	Patrols Needed	Rounded
F-15C	4.19	3.41	4
F-16C	3.74	4.38	5
F-18C	2.63	9.68	10
F-22 30/35	2.76	6.20	7

For the first example, we assume our CAP formation is required to be 500 nautical miles (nmi) ahead of our tanker and each patrol takes 20 minutes to refuel at the tanker. In this notional example, we have a fixed mission distance, and we are looking for the number of aircraft needed to fly this mission. Using equation 5, the number of patrols needed to fully cover is summarized in table 2. We see from this example that there is a clear difference in the number of aircraft required to perform this mission. Assuming patrols were traditionally defined as a two-ship formation, only 8 F-15Cs would be needed to maintain continuous coverage while 10, and 20, and 14 aircraft would be needed for the F-16C, F-18C and F-22 Block 30/35 respectively.

Alternatively, there may be a fixed number of available aircraft to fly this mission, and we need to find the maximum distance we can fly our CAP ahead of our tanker. Using equation 8, we can quickly solve this problem. Looking at each individual aircraft type and for a varying number of available aircraft, we can see the maximum distances we can fly shown in Table 3.

Table 3: Aircraft Maximum Distance

Aircraft	Patrols	CAP Distance from Tanker (nmi)
F-15C	3	380.54
	4	630.54
	5	780.54
F-16C	3	268.67
	4	452.00
	5	562.00
F-18C	3	155.00
	4	280.00
	5	355.00
F-22 30/35	3	190.96
	4	340.69
	5	430.53

4.2 Simulation of Agile Combat Employment in the Spratly Islands of Many Combat Air Patrols

The Spratly Islands, located in the South China Sea, represent one of the most contested regions in in the world, and serves as a focal point for Great Power competition This archipelago, comprising over 100 small islands, reefs, and atolls, is

Figure 6. Spratly Islands Area Map

strategically significant due to its rich natural resources, as well as its critical position along major maritime trade routes. The islands are claimed in whole or in part by several countries, including China, Vietnam, the Philippines, Malaysia, and Taiwan, each asserting historical and legal rights over the territory. A map of the area is included in Figure 6. China's extensive land reclamation and militarization efforts in the Spratly Islands have heightened tensions, drawing international attention and concern over freedom of navigation and regional stability. The land area encompasses an area of about 175,000 square miles.

The notional DCA scenario we evaluate involves a squadron of F-22s deployed half to Singapore and half to Davao, Philippines to provide DCA CAPs to protect this critical maritime route and protect ships from any Chinese aggression. The tankers refueling this mission are considered high value airborne assets, thus the DCA formations primary mission will be to protect the tankers. GPC planning scenarios are inherently more complex and require additional effort and time. For this example, we want to generate F-22 Block 30/35 DCA sorties to protect the tankers over the Spratly Islands. The aerial refueling will take place 500 nautical miles away from Singapore and the CAP location will be 200 nautical miles ahead of the tanker as pictured in Figure 7.

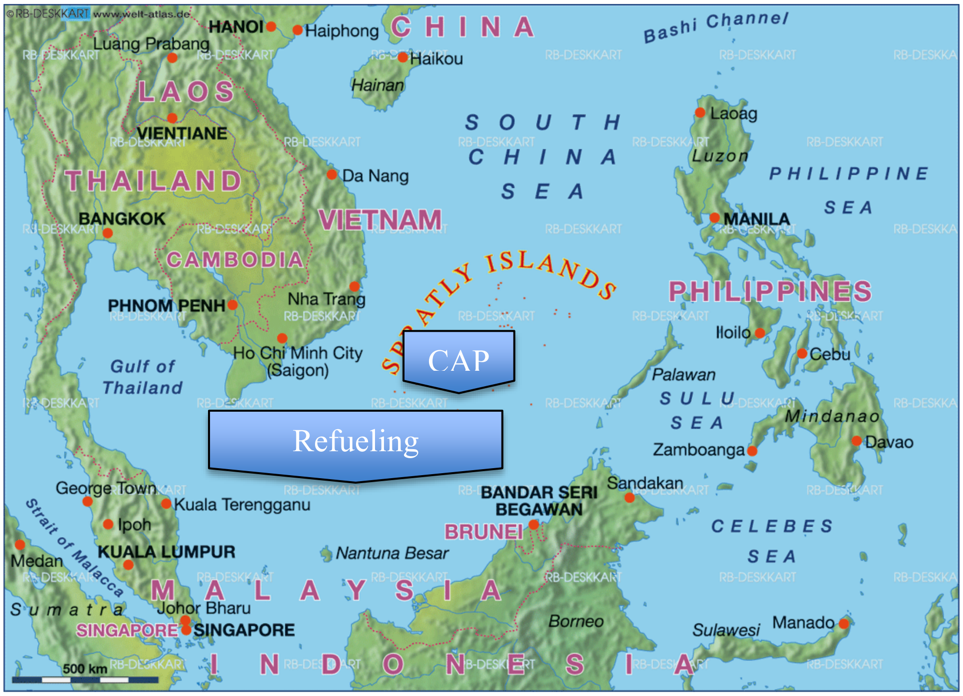

Figure 7. Spratly Islands DCA Map

Mission planning consists of two phases: 1) Force allocation as directed by the joint forces command team, e.g., the number of aircraft and tankers available to achieve the mission and 2) The mission planning of the ATOs, i.e., how to fly the available assets to achieve the desired results. There are typically forty-eight hours between mission planning and execution. Mission planning cells are typically over twenty people bringing together a myriad of information into an executable plan to include but not limited to weather, intelligence, and operator specific tactical data. After talking with subject matter experts, there is variability in how much of that time is spent on calculation. We estimate about four man-hours of the air task order planning is dedicated to mission calculations. We simulated DCA planning times for F-22 sorties as normally distributed with a mean planning time of four hours and a standard deviation of one hour. It would be unrealistic to have planning scenario calculations that took less than an hour, so we truncated the normal distribution at one hour. We refer to the current planning time as the status quo simulation. Our proposed planning time is much faster than the status quo. The solution is instantaneous with the correct inputs; however, we believe there will still be variability in obtaining the appropriate data, e.g., knowing how many aircraft are available for this mission or understanding the mission requirements for distancing.

We simulate planning times with these new equations uniformly between 0.5 and 1.5 hours to account for the variability in obtaining the required information. After running one thousand replications, we find that on average there is a 75.8% time savings. When analyzing the best-case scenario, our proposed planning method still has a 49.9% time savings while the worst-case scenario maintains a 78.6% time savings. The histogram of the status quo, and updated mission planning methods are shown in Figure 8 and Figure 9. Figure 10 shows a sorted plot of the simulated outputs of both the status quo as well as the proposed planning scenario.

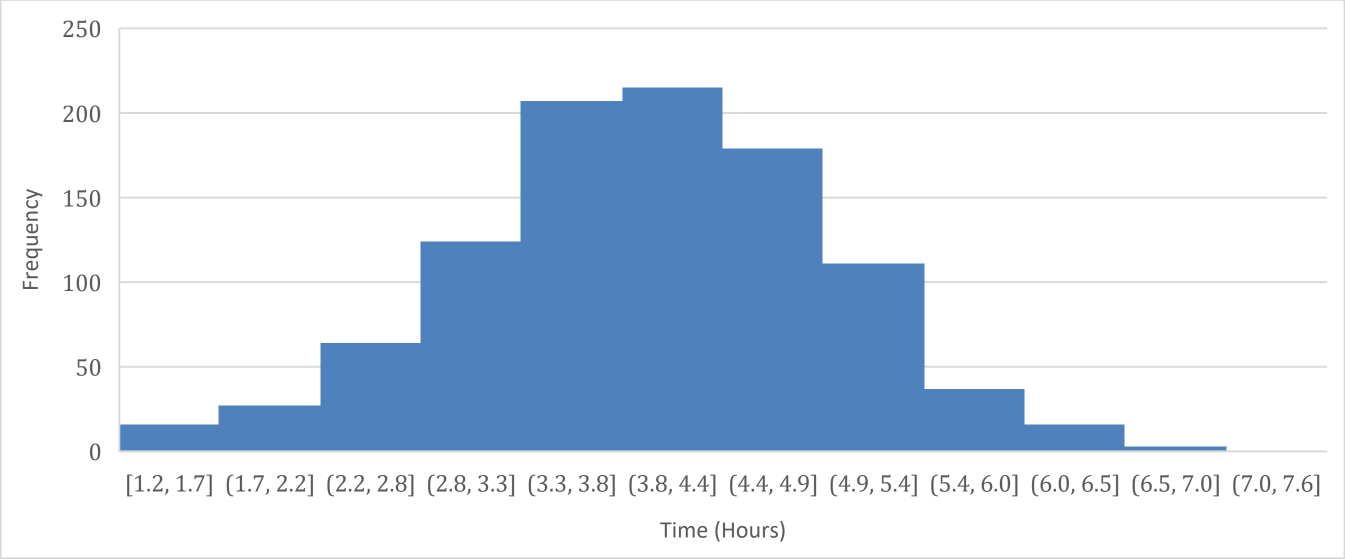

Figure 8. Status Quo Mission Planning Calculation Time Histogram

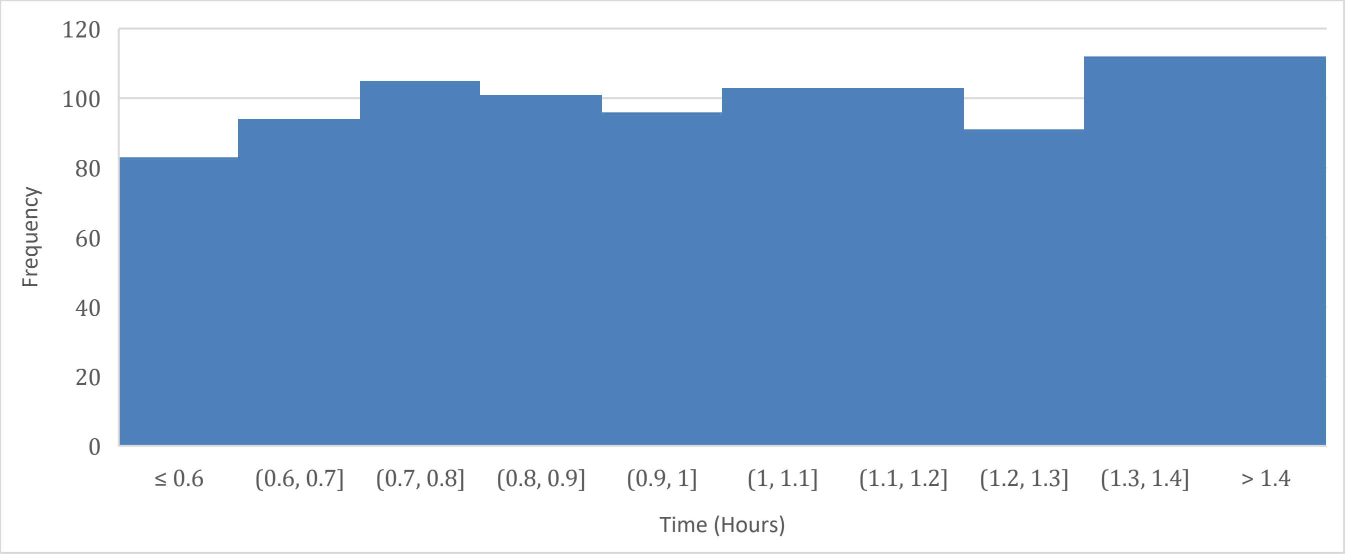

Figure 9. Proposed Mission Planning Calculation Time Histogram

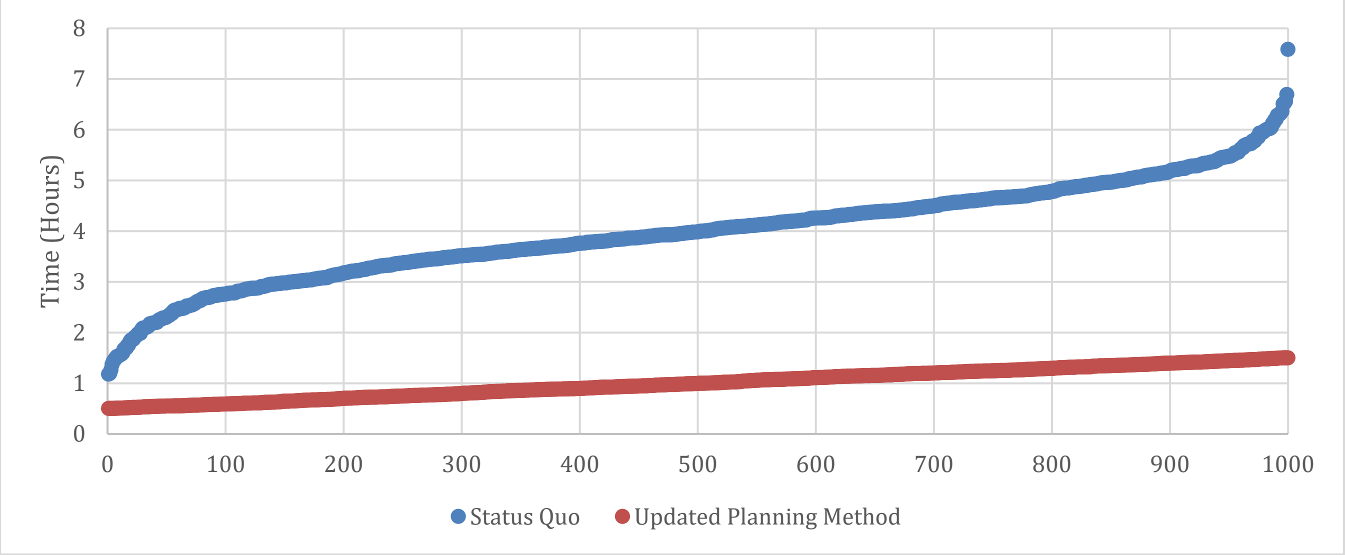

Figure 10. Sorted Simulated Mission Planning Calculation Times

ATO mission planning operates within a critical 48-hour window, where decisions can significantly impact the safety of operators and those under their protection. While our methodology does not shorten this timeframe, it offers substantial efficiency gains, potentially freeing up an average of three hours daily for mission planners. This additional time can be invaluable for refining and optimizing ATO plans. The automation of manual calculations we propose has the potential to dramatically enhance mission planning effectiveness. Furthermore, our equations provide crucial insights into aircraft limitations, which can inform the joint forces command staff's resource allocation decisions prior to publishing ATO asset lists. This improved understanding of operational constraints can play a vital role in developing more effective and realistic asset allocation strategies for mission planners. Ultimately, these enhancements in the planning process can lead to more robust, adaptable, and successful military operations in complex environments.

5. Conclusions and Future Research

As the landscape of Great Power competition evolves, particularly China's rapid military advancements, the United States Air Force must adapt its strategies to maintain air superiority and protect its assets. The ACE concept represents a critical shift in our approach to power projection and homeland defense. This paper's contribution to DCA mission planning aligns closely with the ACE framework, offering a flexible and scalable methodology for rapid decision-making in contested environments. The closed-form equations we've introduced for DCA missile defense scenarios provide an invaluable tool for mission planners operating within the ACE paradigm, and for normal ATO construction. These equations, easily repeatable and instantaneously calculated with the correct inputs, enable planners to quickly assess and adapt to changing threat landscapes. This agility is crucial when facing the evolving missile capabilities of peer adversaries like China, as highlighted in our analysis of missile threat constraints on viable basing options.

An area for possible future research includes incorporating our solution methodology into the ACE site-selection process by offering a means to optimize DCA operations within chosen basing clusters. By integrating our approach with geographic information system analysis, planners could simultaneously evaluate potential ACE locations and optimize DCA patrol configurations. This synergy would allow for a more comprehensive and nuanced approach to mission planning, considering both geographical constraints and operational requirements.

The flexibility of our model aligns with the ACE concept's need for adaptability. Just as the ACE framework allows planners to adjust criteria and constraints based on emerging knowledge or changes in resource availability, our DCA planning tool can be easily modified to account for different aircraft types, mission-ready aircraft numbers, CAP distances, and patrol configurations. This adaptability is crucial in right-of-boom environments, where rapid response to changing conditions is paramount. As we continue to develop and refine our DCA planning efforts, the integration of operations research tools like those presented in this paper will serve as force multipliers in our homeland defense strategy. By leveraging these advanced decision-making methods, we can navigate the complicated scenarios presented by modern warfare and the ongoing Great Power competition.

In conclusion, our research contributes significantly to enhancing the Joint Force's capability to plan and execute flexible, resilient operations in contested environments. As we face potential conflicts in disputed territories, these tools will be essential in adapting our strategies and maintaining global stability. As we face the prospect of potential conflicts in disputed territories, these tools will be essential in adapting our strategies and maintaining global stability.

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