A Mathematical Model for In-Person Office Assignment During COVID-19

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The author would like to thank Ingyu Lee (Graduate Student) and Sangwon Yoon (Associate Professor) for assistance in solving the model.

Abstract: The global pandemic that is COVID-19 has altered our world as we know it. Educational systems have been seriously affected, businesses – whether "essential" or not – have been severely stressed, how people socialize has, perhaps forever, been changed, and telecommuting is the new norm. The purpose of this paper is to discuss a situation wherein a company was having difficulty in scheduling their in-person office staffing in a move to allow rotational schedules for their employees to decongest their office when an entirely telecommuting operation was not possible. Due to the different types of employees, their various work-related constraints, and the need to minimize the number of bodies in the office while addressing minimum coverage necessary to perform the company's daily activities, a quick solution to their scheduling/assignment needs was not obvious or trivially obtained. As such, mathematical models, specifically, integer programming assignment models, were developed and ultimately solved using the Python/Gurobi solver to address their scheduling needs. This paper will describe the constraints faced by the company and the models developed to solve their tricky assignment problem.

Keywords: COVID-19, Workforce Planning, Mathematical Modeling

1. Introduction/Motivation

This paper describes a solution for a company that was having difficulty in establishing a rotating in-person versus telecommuting schedule for their employees in the time of COVID-19. The company is a small insurance brokerage firm, that was(is) considered an essential business. Their office is located in one of the largest US cities. There are different employee types, as will be delineated in Section 2.

The company must have an office presence in order to support customers that chose to visit them in-person; however, they also wanted to decongest the office in order to allow for social distancing. This problem is not entirely different from a set covering problem, such as one found in Bates et al. (2015), and even though those types of problems are NP-Hard, due to the size of the problem studied in this work, optimal results can be found in quick time.

This organization is very good at what they do, but what they do not have expertise in, is Operations Research (OR) level expertise, namely mathematical modeling, for certain areas of operations management. As such, they were struggling to develop their in-person versus telecommuting personnel, and the mathematical model described herein allowed them to solve their problem.

DOI: https://doi.org/10.37266/ISER.2021v9i1.pp68-74

2. Problem Description

This company was struggling to assign in-person versus telecommuting personnel under two primary considerations: 1) minimum in-office coverage, by employee type, to maintain operations and 2) development of a two-week employee assignment that would be cycled every two weeks. In addressing both of the considerations, the company wanted to provide equity among who was assigned in-office presence versus who was allowed to telecommute. As earlier noted, this company was(is) considered an essential company due to the nature of their business. As such, they were allowed and needed to have in-office personnel to address customers that visited in-person.

This company has multiple managers. Each manager has a team consisting of two different employee types, namely, coordinators and assistants. This effort focused on the scheduling of one of the manager's teams within the organization. This manager's team consisted of ten (10) coordinators (C) and three (3) assistants (A).

3. Mathematical Modeling

This model and the solution was presented, but not published at a virtual conference (Santos (2020)). At its essence, this is an OR model. OR is devoted to solving real-world decision-making problems by decomposing them into the following three constituents (Taha, 2003; Winston 2004): 1) identifying the alternatives (e.g., decision variables); 2) identifying the constraints (boundaries) of the problem; and 3) identifying the objective function. The steps in the development of the mathematical model will be presented in this order (in Sections 3.1-3.4):

- Pre-existing (fixed) Constraints
- Rotational (weekly) Constraints
- Objective Function
- Decision Variables

3.1 Pre-existing (fixed) Constraints

Prior to COVID-19 concerns, the company already had some pre-existing constraints that governed whether an employee was in the office or not. Most of the staff worked 9 out of 10 workdays (in a two-week, M-F period). They either took one Monday or one Friday off in a two-week window. The company referred to this as EOMF (every other Monday or Friday (off)). Some of them opted for a Monday off and some of them opted for a Friday off. The new rotational (in-person versus telecommuting) schedule as a result of COVID-19 did not alter the pre-existing EOMF schedule for this team.

If an employee opted out of the first Monday (in a two-week period), they referred to this as EOM1, if an employee opted out of the second Monday, this was referred to as EOM2. It follows that EOF1 and EOF2 refers to employees that opted out of the first Friday or second Friday, respectively, in a two-week window.

Based on the above definitions, the pre-existing EOMF schedule for this team was the following:

- EOM1 – 1 Assistant, 3 Coordinators (1A, 3C)
- EOM2 – 1A, 2C
- EOF1 – 0A, 2C
- EOF2 – 1A, 1C

If one were to count all of the As and Cs in the above, they would obtain 3 As and 8 Cs. As earlier noted, this team has 3 As and 10 Cs, thus there were two coordinator exceptions in the EOMF schedule. One of the coordinators does not opt for EOMF but always remotely works Wednesday and Thursday. Another coordinator gets off every Friday.

3.2 Rotational (weekly) Constraints

The EOMF constraints are, arguably, weekly constraints. And while they are pre-existing (pre-COVID) constraints, there comes a dilemma. The dilemma is to satisfy the employee EOMF constraints while at the same time ensuring a minimum number of each employee type in the office for business coverage.

One solution that the curious reader may be thinking is this: *Why can't everyone just telecommute (and keep their existing EOMF schedule (i.e., not work/telecommute on their EOMF day))?* This is not feasible because there has to be a minimum coverage of warm-bodies in the office. There also needs to be some equitable distribution of who is assigned in-office duties versus telecommuting.

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The minimum in-office coverage by employee type is the following (on any given day):

- There must be at least two coordinators and no more than three
- There must be at least one assistant but no more than two

In addition to the per-day in-person employee type as noted above, there was also discussion about what was considered an equitable schedule, per employee, on having them come into the office in a two-week window. The discussion resulted in this constraint per employee in the two-week window:

- Each coordinator was required to go in at least once, but no more than three times
- Each assistant was also required to go in at least once, but no more than three times

3.3 Objective Function

As will be noted later, after the decision variables are presented, the objective function in this work is really just a "dummy" objective function. The true solution will be to satisfy all of the aforementioned, albeit generalized, constraints.

3.4 The Model

Prior to the presentation of the model, it is prudent to define the decision variables. All of the decision variables are binary and pertain to whether or not an assistant (Equation 1) or a coordinator (Equation 2) is in the office on a given day.

$$A(k,j) = \begin{cases} 1 & Assistant\ k\ works\ in-office\ day\ j \\ 0 & otherwise \end{cases} \quad (1)$$

$$C(i,j) = \begin{cases} 1 & Coordinator\ i\ works\ in-office\ day\ j \\ 0 & otherwise \end{cases} \quad (2)$$

In the above, k=1..3 (as there are three assistants) and i=1..10 (as there are ten coordinators). In each equation, j=1..10, where j=1 refers to the first Monday, j=2 refers to the first Tuesday, … and j=10 refers to the second Friday.

3.4.1 Generalized Constraints in Model Form

What is to follow now are the generalized constraints that will satisfy the per-day in-person constraints by employee type, and the bi-weekly constraints to constrain the number of in-person visits, per employee type. The EOMF constraints will be presented in the full model in the Appendix.

There must be at least two coordinators in the office every day, but no more than three of them, is generalized as Equation 3.

$$2 \leq \sum_i C_{i,j} \leq 3 \quad \forall j \quad (3)$$

There must be at least one assistant in the office every day, but no more than two, is generalized as Equation 4.

$$1 \leq \sum_k A_{k,j} \leq 2 \quad \forall j \quad (4)$$

Each coordinator is in the office [1,3] times (inclusive) in each two weeks is generalized as Equation 5.

$$1 \leq \sum_j C_{i,j} \leq 3 \quad \forall i \quad (5)$$

Each assistant is in the office between [1,3] times (inclusive) in each two-week span is generalized as Equation (6).

$$1 \leq \sum_j A_{k,j} \leq 3 \quad \forall k \quad (6)$$

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3.4.2 The Complete Model

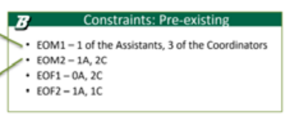
The complete model is presented as Figure 1. It is presented as if it were to be solved using LINDO™. In LINDO™, an exclamation point (!) is indicative of a comment statement. As noted earlier, a dummy objective function is used. The dummy objective function chosen is this: "Minimize C11". Regardless of the final value (0 or 1) of C11, any solution that satisfied all constraints is feasible and addresses the company's concerns and solved their assignment problem. Albeit, it is true that if there are multiple solutions where Coordinator 1 is (C11=1) or is not (C11=0) in the office in the first Monday of each two-week period, the model will pick the one where C11=0.

```
Minimize C11

s.t.

! At least 2 Coordinators in the office every day but no more than 3 Coordinators

C11 + C21 + C31 + C41 + C51 + C61 + C71 + C81 + C91 + C101 >= 2
C12 + C22 + C32 + C42 + C52 + C62 + C72 + C82 + C92 + C102 >= 2
C13 + C23 + C33 + C43 + C53 + C63 + C73 + C83 + C93 + C103 >= 2
C14 + C24 + C34 + C44 + C54 + C64 + C74 + C84 + C94 + C104 >= 2
C15 + C25 + C35 + C45 + C55 + C65 + C75 + C85 + C95 + C105 >= 2
C16 + C26 + C36 + C46 + C56 + C66 + C76 + C86 + C96 + C106 >= 2
C17 + C27 + C37 + C47 + C57 + C67 + C77 + C87 + C97 + C107 >= 2
C18 + C28 + C38 + C48 + C58 + C68 + C78 + C88 + C98 + C108 >= 2
C19 + C29 + C39 + C49 + C59 + C69 + C79 + C89 + C99 + C109 >= 2
C110 + C210 + C310 + C410 + C510 + C610 + C710 + C810 + C910 + C1010 >= 2

C11 + C21 + C31 + C41 + C51 + C61 + C71 + C81 + C91 + C101 <= 3
C12 + C22 + C32 + C42 + C52 + C62 + C72 + C82 + C92 + C102 <= 3
C13 + C23 + C33 + C43 + C53 + C63 + C73 + C83 + C93 + C103 <= 3
C14 + C24 + C34 + C44 + C54 + C64 + C74 + C84 + C94 + C104 <= 3
C15 + C25 + C35 + C45 + C55 + C65 + C75 + C85 + C95 + C105 <= 3
C16 + C26 + C36 + C46 + C56 + C66 + C76 + C86 + C96 + C106 <= 3
C17 + C27 + C37 + C47 + C57 + C67 + C77 + C87 + C97 + C107 <= 3
C18 + C28 + C38 + C48 + C58 + C68 + C78 + C88 + C98 + C108 <= 3
C19 + C29 + C39 + C49 + C59 + C69 + C79 + C89 + C99 + C109 <= 3
C110 + C210 + C310 + C410 + C510 + C610 + C710 + C810 + C910 + C1010 <= 3

! At least one Assistant in the office every day, but no more than 2

A11 + A21 + A31 >= 1
A12 + A22 + A32 >= 1
A13 + A23 + A33 >= 1
A14 + A24 + A34 >= 1
A15 + A25 + A35 >= 1
A16 + A26 + A36 >= 1
A17 + A27 + A37 >= 1
A18 + A28 + A38 >= 1
A19 + A29 + A39 >= 1
A110 + A210 + A310 >= 1

A11 + A21 + A31 <= 2
A12 + A22 + A32 <= 2
A13 + A23 + A33 <= 2
A14 + A24 + A34 <= 2
A15 + A25 + A35 <= 2
A16 + A26 + A36 <= 2
A17 + A27 + A37 <= 2
A18 + A28 + A38 <= 2
A19 + A29 + A39 <= 2
A110 + A210 + A310 <= 2
```

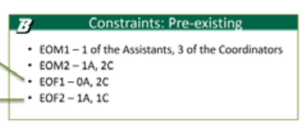
(Figure 1 to be continued)

DOI: https://doi.org/10.37266/ISER.2021v9i1.pp68-74

```
! Each Coordinator in the office [1,3] times (inclusive) each 2 weeks

C11 + C12 + C13 + C14 + C15 + C16 + C17 + C18 + C19 + C110 >=1
C21 + C22 + C23 + C24 + C25 + C26 + C27 + C28 + C29 + C210 >=1
C31 + C32 + C33 + C34 + C35 + C36 + C37 + C38 + C39 + C310 >=1
C41 + C42 + C43 + C44 + C45 + C46 + C47 + C48 + C49 + C410 >=1
C51 + C52 + C53 + C54 + C55 + C56 + C57 + C58 + C59 + C510 >=1
C61 + C62 + C63 + C64 + C65 + C66 + C67 + C68 + C69 + C610 >=1
C71 + C72 + C73 + C74 + C75 + C76 + C77 + C78 + C79 + C710 >=1
C81 + C82 + C83 + C84 + C85 + C86 + C87 + C88 + C89 + C810 >=1
C91 + C92 + C93 + C94 + C95 + C96 + C97 + C98 + C99 + C910 >=1
C101 + C102 + C103 + C104 + C105 + C106 + C107 + C108 + C109 + C1010 >=1

C11 + C12 + C13 + C14 + C15 + C16 + C17 + C18 + C19 + C110 <=3
C21 + C22 + C23 + C24 + C25 + C26 + C27 + C28 + C29 + C210 <=3
C31 + C32 + C33 + C34 + C35 + C36 + C37 + C38 + C39 + C310 <=3
C41 + C42 + C43 + C44 + C45 + C46 + C47 + C48 + C49 + C410 <=3
C51 + C52 + C53 + C54 + C55 + C56 + C57 + C58 + C59 + C510 <=3
C61 + C62 + C63 + C64 + C65 + C66 + C67 + C68 + C69 + C610 <=3
C71 + C72 + C73 + C74 + C75 + C76 + C77 + C78 + C79 + C710 <=3
C81 + C82 + C83 + C84 + C85 + C86 + C87 + C88 + C89 + C810 <=3
C91 + C92 + C93 + C94 + C95 + C96 + C97 + C98 + C99 + C910 <=3
C101 + C102 + C103 + C104 + C105 + C106 + C107 + C108 + C109 + C1010 <= 3

! Each Assistant in the office between [1,3] times (inclusive) in 2 week span

A11 + A12 + A13 + A14 + A15 + A16 + A17 + A18 + A19 + A110 >=1
A21 + A22 + A23 + A24 + A25 + A26 + A27 + A28 + A29 + A210 >=1
A31 + A32 + A33 + A34 + A35 + A36 + A37 + A38 + A39 + A310 >=1

A11 + A12 + A13 + A14 + A15 + A16 + A17 + A18 + A19 + A110 <=3
A21 + A22 + A23 + A24 + A25 + A26 + A27 + A28 + A29 + A210 <=3
A31 + A32 + A33 + A34 + A35 + A36 + A37 + A38 + A39 + A310 <=3

! EOM1 Constraints

A11 = 0
C31 = 0
C61 = 0
C101 = 0

! EOM2 Constraints
A36 = 0
C26 = 0
C86 = 0
```




```
! EOF1 Constraints

C45 = 0
C75 = 0

! EOF2 Constraints

A210 = 0
C510 = 0
```



(Figure 1 to be continued)

```
! Coordinator that works at home W&Th

C13 = 0
C14 = 0
C18 = 0
C19 = 0

! Coordinator not in the office on any Friday

C95 = 0
C910 = 0


END
```

2 Exceptions to Above:
– One of the Coordinators does not opt for EOMF but always remotely works Wednesday and Thursday
– One of the Coordinators gets off every Friday

All variables are declared to be binary

Figure 1. The Complete Mathematical Model

4. Solution

The model was originally set up to be solved using LINDO™ or CPLEX. However, the office wherein the author has the computer with those solvers residing on the hard drive, was inaccessible because, ironically enough, of COVID restrictions. Fortunately, there was a research team that was allowed to be on campus due to the nature of their research. A professor and a graduate student from that team solved the model for the author. They utilized Python with the Gurobi solver.

The first attempt at a solution was infeasible, which turned out to be obvious. For the assistants, they need at least one in the office per day. If each of the three assistants can only go to the office a maximum of three times in the ten day window, this accounts for nine assistant-days when they need a minimum of 10 assistant-days. The manager identified an assistant that would be willing to go to the office more than three times in a ten day period. Once that constraint (the third assistant's) was modified (to be <= 4), a feasible solution was found (Table 1). In the table, the rows represent the individual coordinator or assistant, and the columns represent the ten days of the two-week window. A "1" indicates the employee is in-office that day, a "0" indicates they are not in-office that day. For example, the reader will note that Assistant 3 is in-office 4 times in the two-week window.

Table 1. Solution to the Two-Week Rotation Schedule

C_{ij}	1	2	3	4	5	6	7	8	9	10
1	0	0	0	0	0	0	1	0	0	1
2	0	0	1	1	0	0	0	0	0	1
3	0	1	0	1	0	0	0	0	0	0
4	1	0	0	0	0	1	0	1	0	0
5	1	0	0	0	0	0	0	0	1	0
6	0	1	0	0	0	1	1	0	0	0
7	0	0	0	0	0	1	0	0	0	0
8	0	0	0	0	1	0	0	1	1	0
9	0	1	0	0	0	0	0	0	0	0
10	0	0	1	0	1	0	0	0	0	0

A_{kj}	1	2	3	4	5	6	7	8	9	10
1	0	0	0	0	0	1	1	1	0	0
2	1	0	1	1	0	0	0	0	0	0
3	0	1	0	0	1	0	0	0	1	1

DOI: https://doi.org/10.37266/ISER.2021v9i1.pp68-74

5. Summary and Conclusions

Oftentimes, we industrial and systems engineers that interface with industrial partners, encounter problems in our domains of expertise, that a company has no idea how to approach the solution. This is exactly the situation, herein. The company has domain expertise in the insurance realm, but was unaware how to solve this type of operations management problem. In fact, when this work was presented at a conference (Santos (2020)), an audience member noted that this could be applied in an organization that they heard was struggling with this type of COVID-19 rotation schedule development.

For the organization in this work, different managers were tasked with assigning their rotational assignments within their teams. This team was the first to solve their scheduling problem, using this approach, and did so in a very quick timeframe. This became a source of pride for the manager that obtained his schedule. All of the coordinators and assistants, with the author being told "especially the assistants," were quite happy with the final assignments.

Other potential areas for utilization of this type of modeling can be suggested as follows:

- As noted for the organization under study in this work, the other managers can develop schedules as based on their number of coordinators and assistants, and incorporating their EOMF schedules.
- The audience member noted above was referring to government offices that faced personnel allocation issues during their in-office COVID scheduling.
- Companies are reducing their physical footprints by cutting space not only in the U.S. (Randall, 2020), but in other countries as well (Allwork.Space-Press, 2020). As such, many different organizations may be faced with the challenge of developing in-person versus telecommuting schedules that can benefit from this type of approach.

The author has also noted instances in higher educational settings where applying OR modeling, particularly but not restricted to integer linear programming (ILP) problems, can assist in decision making. As was discussed at a poster presentation, in support of the Santos (2007) conference paper, the author has used Data Envelopment Analysis (DEA) to compare academic departments, has used ILP to assign judges to evaluate first-year engineering design student projects, and has incorporated capital budgeting models to select from different competing projects.

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DOI: https://doi.org/10.37266/ISER.2021v9i1.pp68-74